COLOR VISION, BRIGHTNESS, RESOLUTION AND CONTRAST IN BINOCULAR IMAGES

A review of published data

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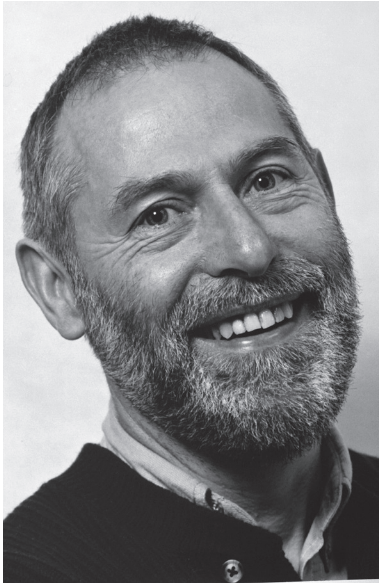
May 2013

Why this document

On several fora (sometimes heated) discussions take place about different aspects of binocular images, how they come about and the occurrence of distortions and their effects. The discussions generated sometimes a rather confusing picture when compared with data found in research literature about these topics. I, therefore, investigated the different aspects of color vision, resolution (sharpness), contrast and the effect of light intensity using published data. The following text is an extract from these data.



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1. Introduction

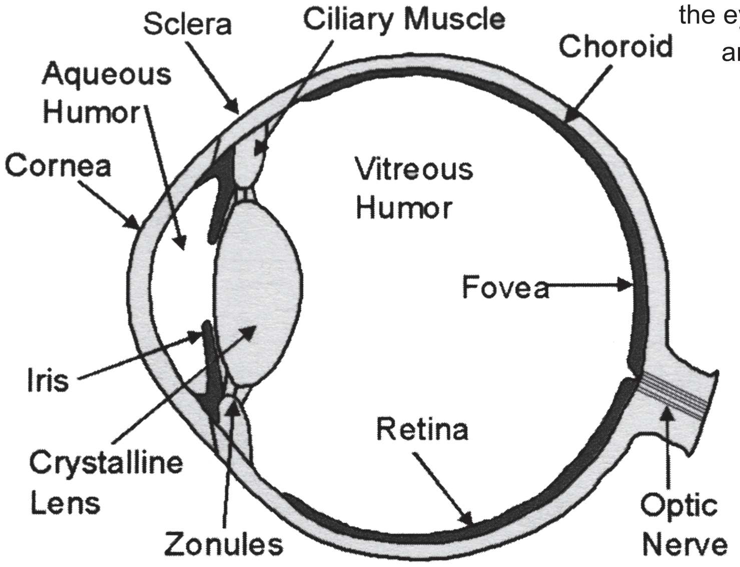
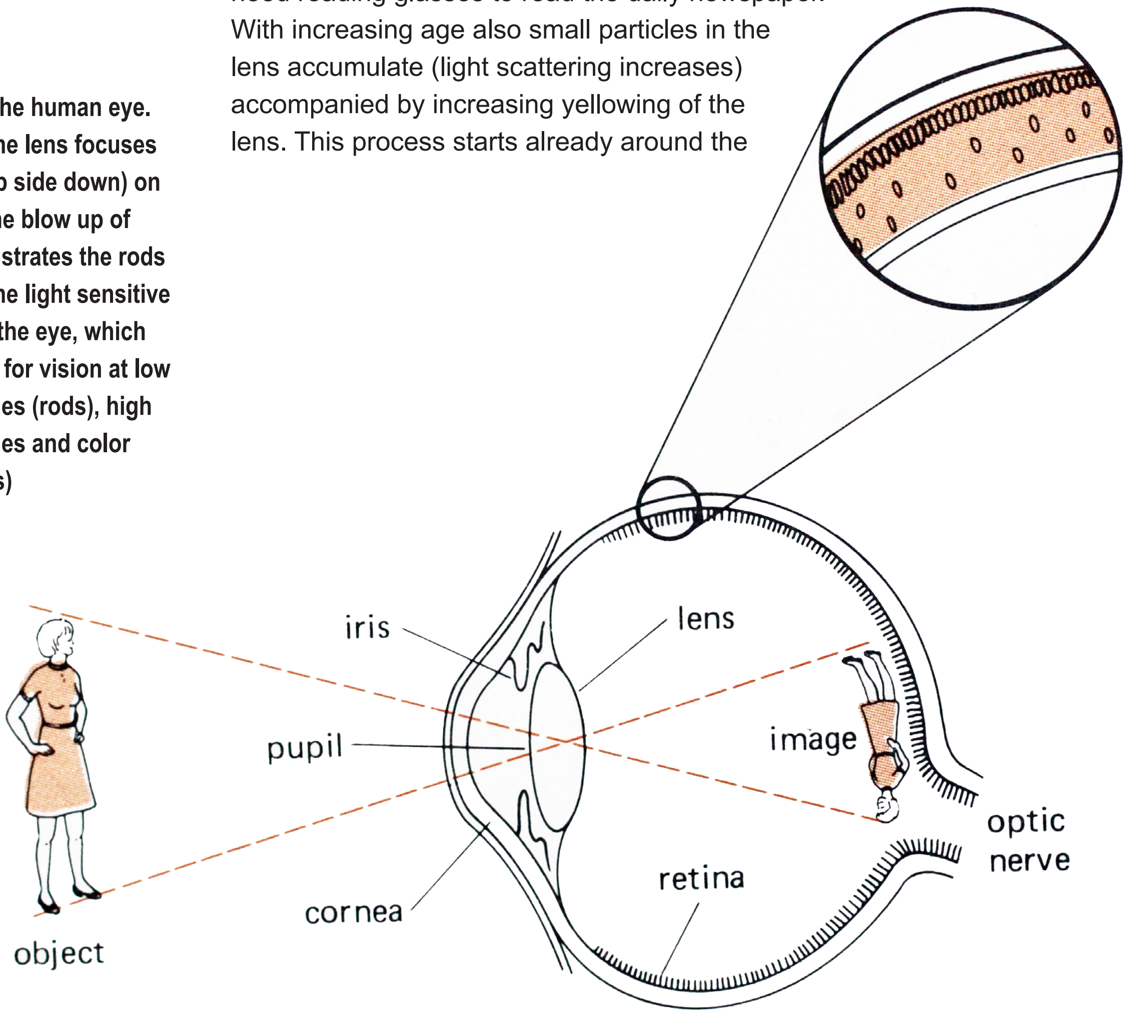


Figure 1
The human eye

The image quality of binoculars has three main players: light, the eye and the optical glass and its coatings in binoculars and telescopes. Image formation in our eyes starts with light entering through the transparent cover called the cornea, which does about 70% of the necessary bending of the light before it passes through the pupil, which is an aperture in the iris. The light then passes through the lens. The function of the lens of the eye is to focus a sharp image of the external world upon its light sensitive layer, the retina. The image projected by the lens on the retina is inverted and upside down. By some means the brain learns early in life to interpret the image of the observed world right side up. In order to focus the eye on both near and distant objects, the shape of the lens is changed by means of the ciliary muscle. When this muscle is totally relaxed, a normal eye will form an image of a distant object on the retina. To generate a sharp image at closer distances the muscular system takes care of shape variations of the lens, so the focal length is changed and a sharp image of a nearby object is formed (accommodation). With increasing age the elasticity of the lens decreases and the muscular system, which functions to change the shape of the lens, hampers and we need reading glasses to read the daily newspaper.

With increasing age also small particles in the lens accumulate (light scattering increases) accompanied by increasing yellowing of the lens. This process starts already around the

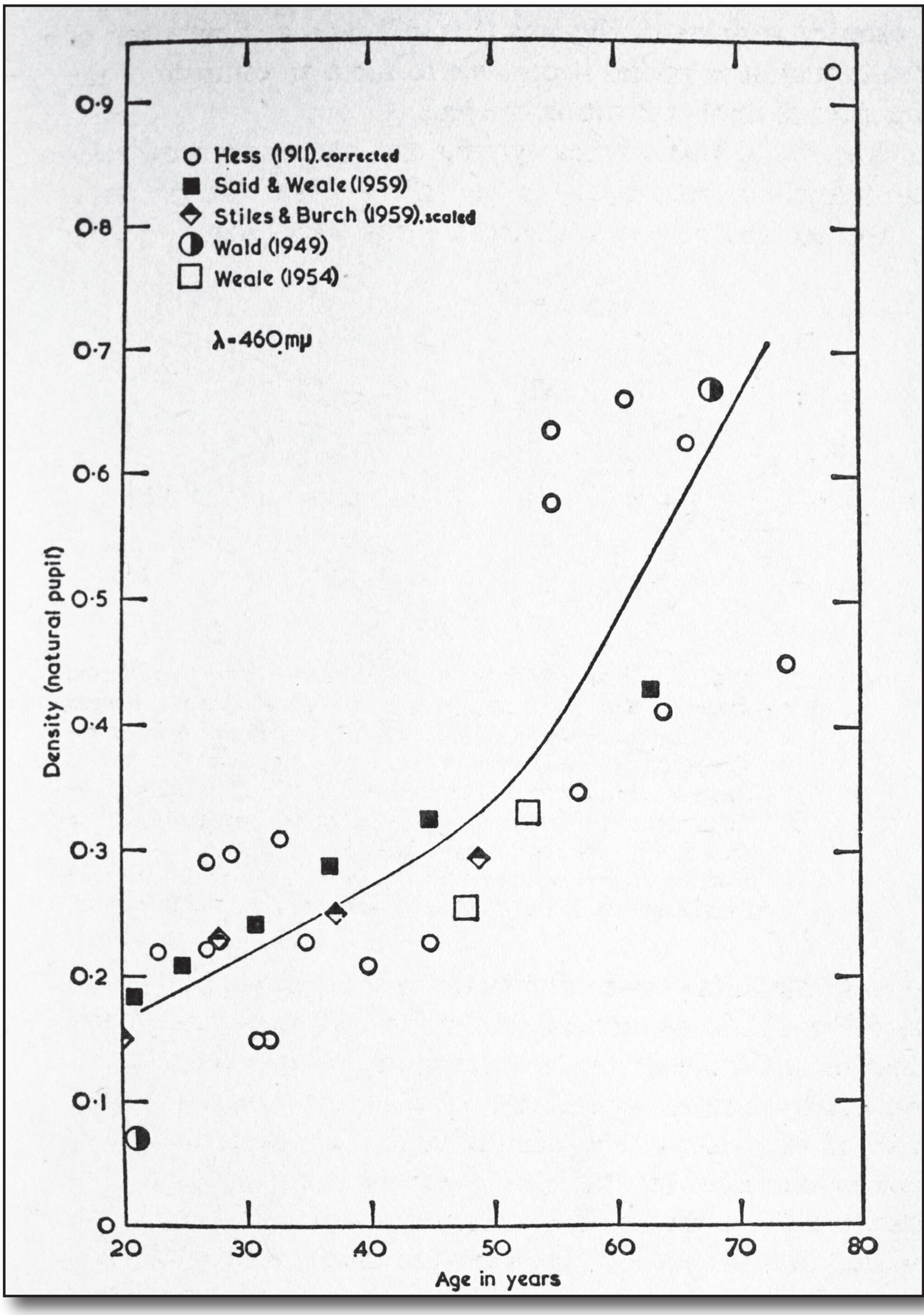
Figure 2
Anatomy of the human eye. The crystalline lens focuses the image (up side down) on the retina. The blow up of the retina illustrates the rods and cones, the light sensitive detectors of the eye, which are essential for vision at low light intensities (rods), high light intensities and color vision (cones)



age of 20 and gets worse and worse with increasing age. As a consequence the lens of the eye absorbs more light and less light reaches the light detection system on the retina of the eye. At the age of sixty 50% or more of the incoming light may be absorbed in the lens through this process and is lost for image formation. The decrease of the amount of light entering the ageing eye is important, since this process affects the visual resolving abilities because eye resolution is strongly influenced by the amount of light available for the eye's light detection system, as I will describe later. To minimize the effects of this ageing process, high light transmissions of binoculars and telescopes are very helpful. High light transmission of binoculars yields images with a higher light output, which are important for optimal visual resolution as I will describe below.

N.B. The neurophysiological system which regulates the color vision process of the eye takes care that the colors reported by the eye-brain system are hardly affected by the yellowing of the eye's lens as I will describe later.

Figure 3
Lenticular yellowing as a function of the age of the eye. The photometric density of the human crystalline lens increases for blue light at first slowly towards the end of the fifth decade, but thereafter at almost four times this rate.



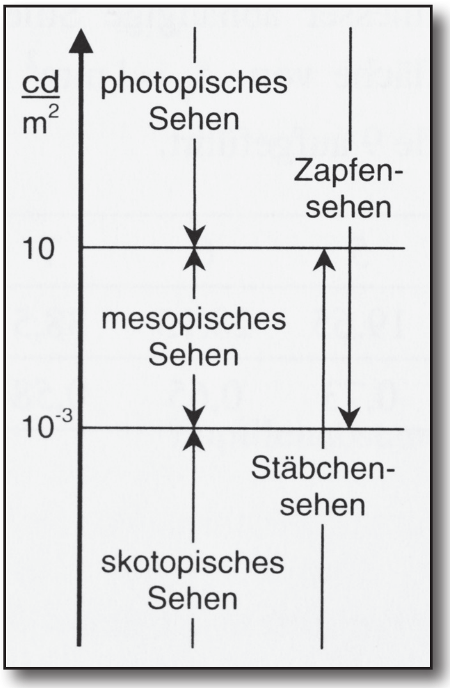
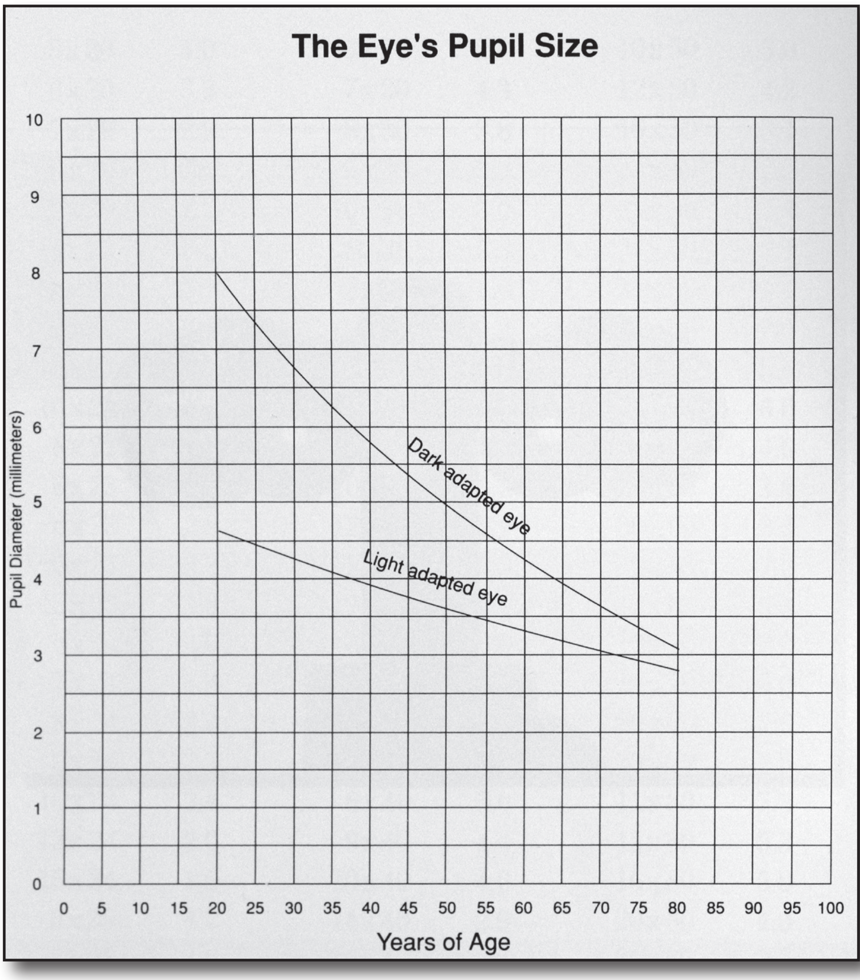


Figure 4
Cone (*Zapfen*) and rod (*Stäpchen*) vision at different light intensities: photopic vision at bright daylight, mesopic vision at twilight and scotopic vision at night. (*above*)

Figure 5
The decrease of the maximum pupil size of the eye with increasing age (*right*)



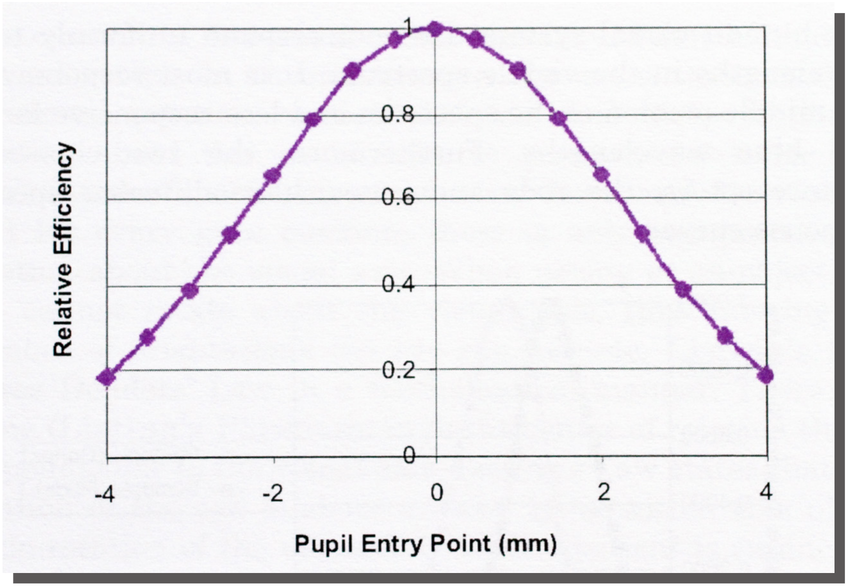
2. The light detection system of the eye

Light enters the eye through the transparent cover called the cornea, which does about 70% of the necessary bending of the light before it passes through the pupil, which is an aperture in the iris. The light then passes through the lens. The function of the lens of the eye is to focus a sharp image of the external world upon its light sensitive layer, the retina. The retina is a curved image surface and the iris in front of the lens acts as an aperture stop of variable diameter. The diameter of the pupil depends on the amount of light the eye is exposed to. At high light intensities the average pupil size is approximately 2 mm in full day light and approximately 8 mm for the fully dark adapted eye. With increasing age the maximum size of the dark adapted pupil decreases from around 8 mm at the age of ten to about 4-5 mm at the age of 70, although there significant variations among individuals of the same age.

2a Pupil size and its inhomogeneous luminosity: Stiles-Crawford factor

It was found by Stiles and Crawford that light entering the eye near the centre of the pupil is usually more effective in producing a visual response than that entering

Figure 6
Stiles-Crawford effect. The curve shows the decrease of the visual response of the eye from the center to the periphery of the pupil



near the periphery. This has been attributed to a directional property of the retinal photoreceptor cells. This led to the introduction of the so-called Stiles-Crawford factor. The effect of it is as follows. If the maximum pupil luminosity is defined as I-max, than a pupil size of 1 mm yields a Stiles-Crawford factor of 1. This means the luminosity of the pupil is homogeneous over the whole area of the pupil, there is no decline of luminosity towards the pupil periphery. That picture changes with increasing pupil size. At a pupil size of 4 mm the actual pupil's luminosity I-real= 0,90 x I-max, where 0,90 is the Stiles-Crawford factor for that pupil size and I-real is the effective luminosity of a 4 mm eye pupil. The Stiles-Crawford factor than steadily decreases from 0,73 for a 5 mm eye pupil to 0,65 for a 6 mm eye pupil and 0,58 for a 7 mm eye pupil.

2b Photoreceptor cels in the retina

The retina is covered with two basic types of photoreceptor cells: rods and cones (their names were originally inspired by the shape of these photoreceptors as observed by microscopic observations). Each eye has about six to seven million cones and more than hundred million rods. All these photoreceptor cells are dispersed nonuniformly over most of the retina. The rods and cones are connected to about one million optic nerve fibers, which relay the light-stimulated signals to the brain. In the region where the optic nerve enters the eyeball, there are no rods or cones. As a result, the eye has a "blind spot" for which there is no optical response.

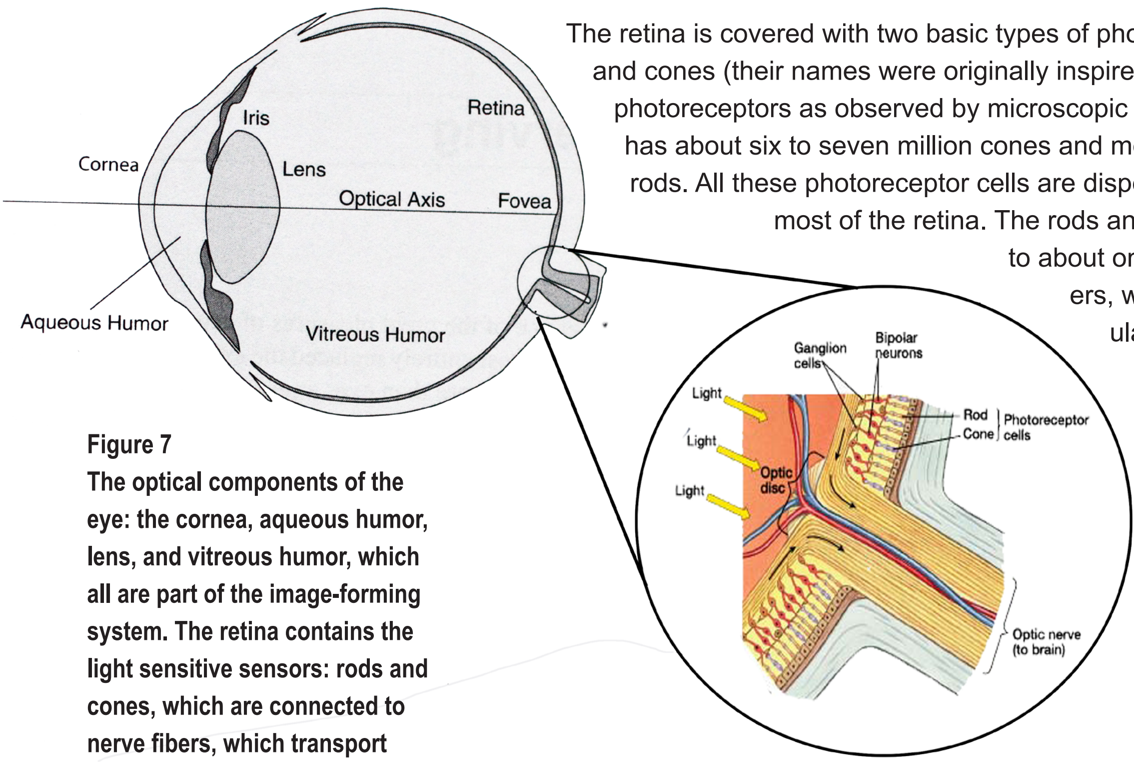


Figure 7
The optical components of the eye: the cornea, aqueous humor, lens, and vitreous humor, which all are part of the image-forming system. The retina contains the light sensitive sensors: rods and cones, which are connected to nerve fibers, which transport the opto-electrical signals to the brain

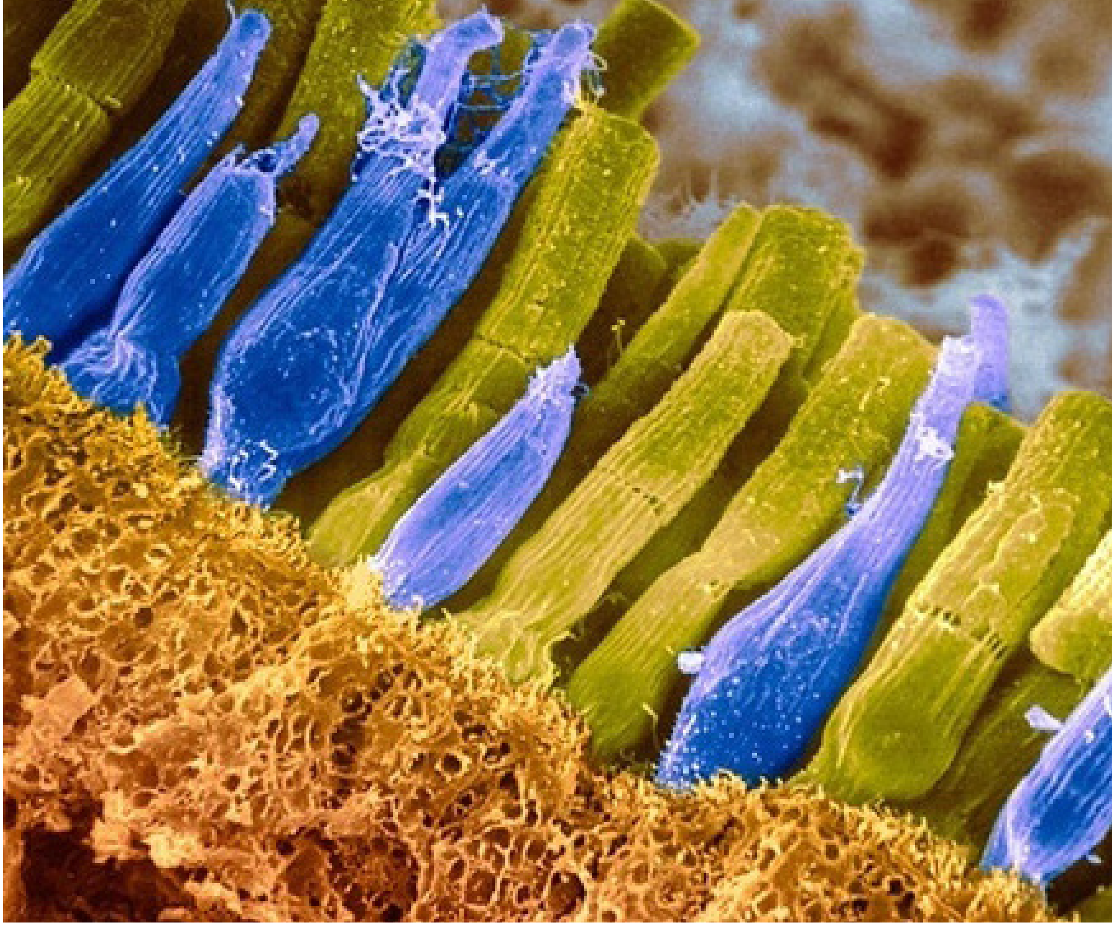


Figure 8
Rods and cones in retina: colourized scanning electron micrograph (SEM) of rods and cones. Rods (green) are long nerve cells which respond to dim light, enabling images to be detected. Cones (blue) are shorter cone-like cells which detect colour.

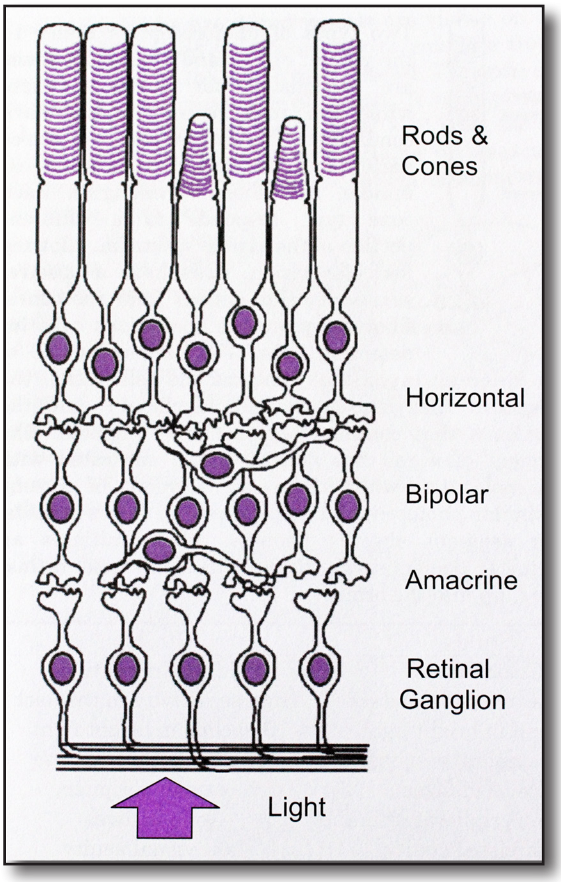


Figure 9
Nerve connections in the retina

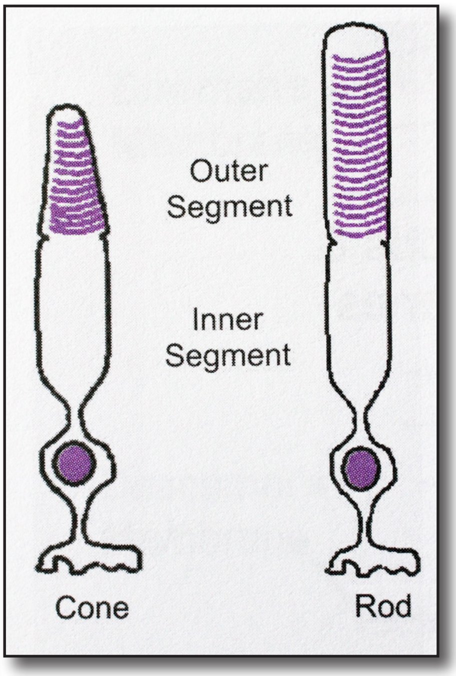


Figure 10
Schematic drawing of rods and cones

The photosensitive material in rods and cones is retinal, a purplish derivative of Vitamin A, bound to a protein with the name opsin. The human eye has four types of retinal-opsin complexes: rhodopsin present in rods and erythrolabe, chlorolabe and cyanolabe present in respectively Blue-sensitive cones, Green sensitive cones and Red sensitive cones. The optical spectra of these light sensitive pigments differ and this difference is the structural basis of night vision and color vision in daylight.

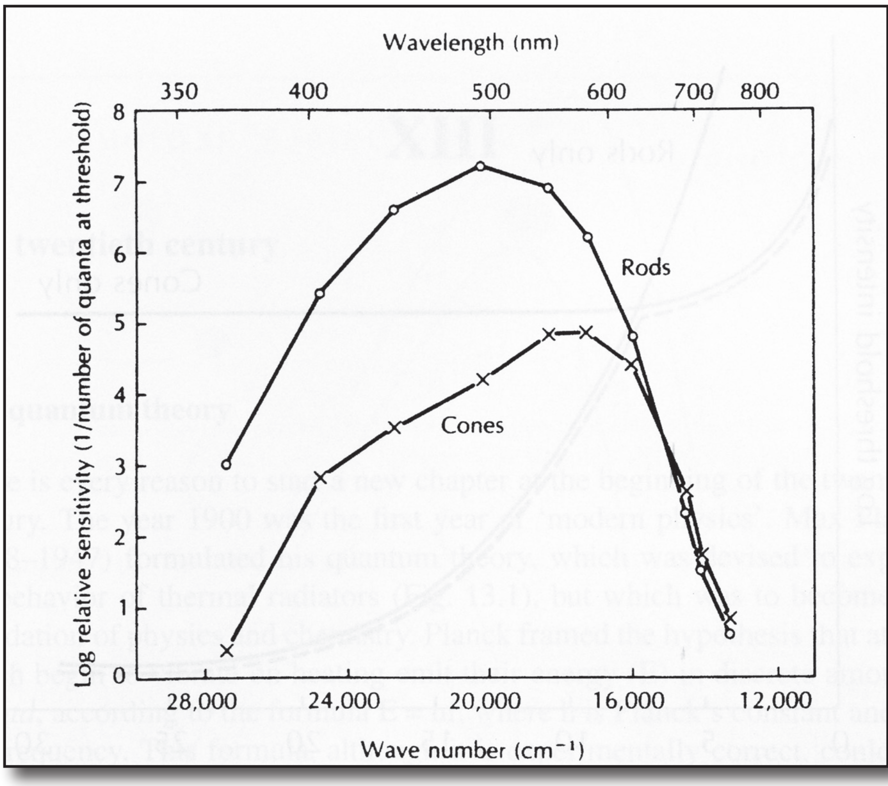
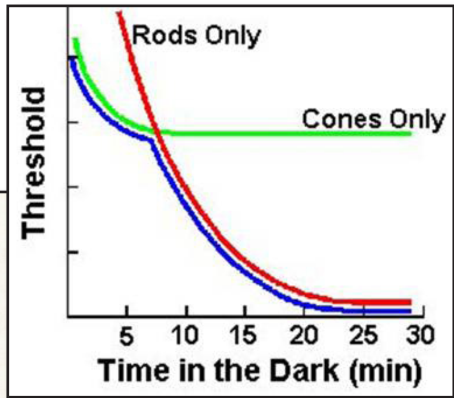


Figure 11
Spectral sensitivity curves in scotopic and photopic vision in a logarithmic scale (after Wald)

When light strikes a molecule of rhodopsin, electron motion causes the molecule to change shape and generate an electrical signal that can be transmitted to the brain. The molecule in its new shape is unstable and decays through a series of steps that result in loss of color or bleaching. Since sensitivity of the photoreceptors depends on the amount of rhodopsin (visual purple) present, it must be regenerated. This regeneration occurs most rapidly in the dark and requires a supply of vitamin A. Lack of Vitamin A retards the regeneration process and leads to night blindness. The regeneration of the photosensitive pigments of cones is a faster process than the regeneration of rhodopsin in rods and as a consequence the so-called adaptation process of the eye to darkness takes almost half an hour due to the slow regeneration process of the rods. The regeneration of the light sensitive pigments in cones is fast



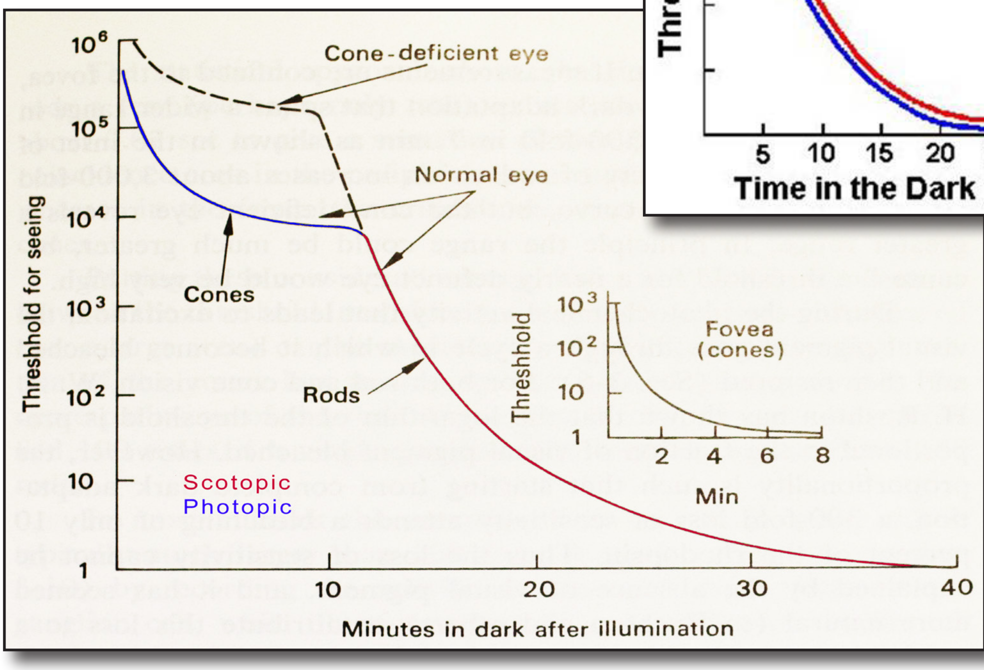


Figure 12
Dark adaptation in human vision. The threshold declines in the dark after a period of illumination; maximum sensitivity is attained after 30-40 minutes. The two branches of the main curve reflect conditions in which the greatest sensitivity is provided by the cones and the rods respectively. Colors can be seen in the tests that define the cone-vision branch, but not under conditions of rod vision. The inset shows dark adaptation in the fovea. The scale of the ordinate in the inset should not be compared with that of the main figure, because the area of illumination in the foveal tests is much smaller.

and takes place when the pigments are not exposed to light during the time span between exposure due to rapid eye movements.

The ensemble of rods (each about 2 micrometers in diameter) resembles in some respects a high-speed black and white film, since the rods are extremely sensitive but they are unable to distinguish color. That has to do with the molecular and spectroscopic properties of the light sensitive pigment rhodopsin present in rods. The absorption maximum of rhodopsin in the rods is at 505-510 nm. Research has shown that the wavelength dependent physiological activity of the rods (measured with so-called action spectra) perfectly matches the absorption spectrum of rhodopsin. This is taken as evidence that the eye has its optimum sensitivity at low light levels at the spectral maximum of rhodopsin absorption around 507 nm. The rods are most suited to perform observations at low light levels like twilight, moonlight, night scenes etc. The photophysical/photochemical property of rhodopsin is the structural basis of the eye for its night vision performance (so-called scotopic vision, where scotopic is derived from the Greek σκοτοσ= dark).

At high light levels rhodopsin is converted into a molecular structure, which does not function as a light collecting pigment and the rods become useless for daylight vision. At lower light levels this inactive structure can be converted back to the light sensitive rhodopsin by an enzymatic reaction (catalytic reaction if you want). That is a rather slow process, so this so-called dark-adaption of the rods takes some time. Therefore your eyes need some time to adjust to darkness. Rods are more sensitive toward the blue end of the color spectrum than cones. On the other hand the cones are sensitive to deep red where the rods are not sensitive at all in that spectral region. Red light may as well be black as far as the rods can tell.

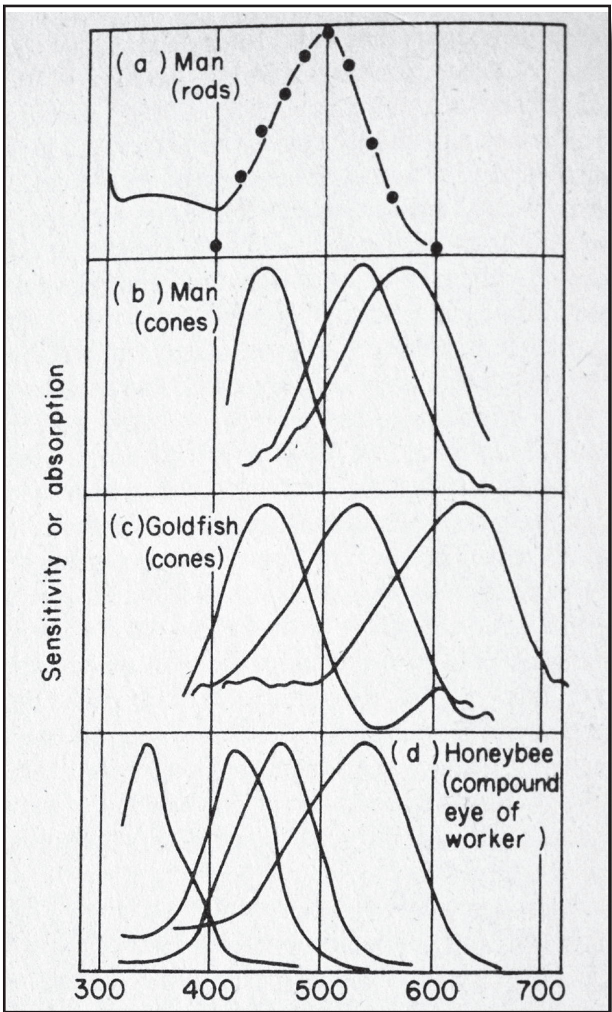


Figure 13
Measured spectra of human visual pigments. (a) = sensitivity or action spectrum for dark adapted retina (dots) and absorption spectrum for rod visual pigment (rhodopsin, solid line). For light having a wavelength of less than 450 nm the sensitivity curve for the complete eye (not shown) is lower than the dotted curve due to light absorption in the lens; this discrepancy increase with increasing age (see also figure 3).
(b) Absorption spectra for the visual pigments in the three types of cones. The absorption maxima are at (from the left) 447 nm for cyanolabe, 540 nm for chlorolabe and 557 nm for erythrolabe. (c) and (d) are resp. absorption spectra for the visual pigments in the three types of cones of the Goldfish and sensitivity spectra for four types of visual cells in the compound eye of the honey-bee.

3. Visual resolution and its structural organisation

In contrast the ensemble of 6-7 million cones (each about 6 micrometers in diameter) can be compared with a low-speed color film. The cones perform in bright light yielding detailed colored views, but they are rather insensitive at low light levels. Cone vision is indicated as daylight vision or photopic vision (photopic comes from the Greek φοτοσ=light).

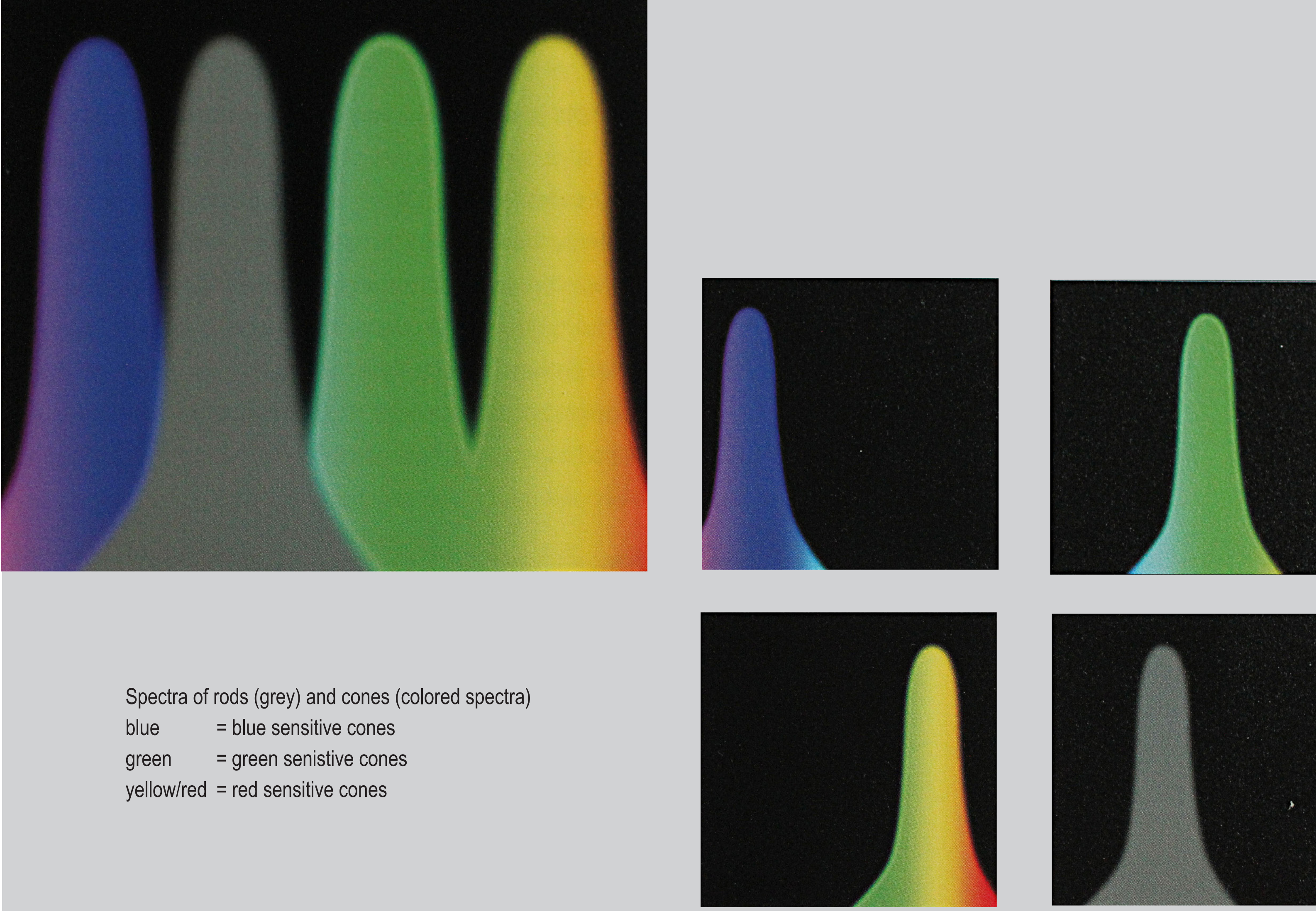
Responsible for the color sensitivity of the cones are three types of cones: a blue sensitive cone population (B) with maximum absorption around 447 nm, a green sensitive cone population (G) with absorption maximum around 540 nm and a red-sensitive cone population (R) with absorption maximum around 585 nm (this wavelength is actually in the yellow spectral region, but for historic reasons it is generally indicated as “red” in connection with this cone population.

(N.B. In vision literature one can find different indications for B, G and R-cones. For example B-cones as β- or S (=Short-wavelenght)-cones, G-cones as γ-or M (=Medium-wavelength)-cones and R-cones as ρ-or L (=Long wavelength)-cones. For reasons of simplicity I have chosen here for the indications B,G and R).

The tail of the absorption spectrum of the “red-sensitive” cones extents into the red part of the spectrum and these cones are sensitive up into the deep red. The concentration ratio of the different cones in the retina is approximately B:G:R = 1:20:40 (in another publication a ratio of 1:16:32 is estimated). The reason for the small number of B-cones is that the eye is not corrected for chromatic aberration, so that blue, green and red (=yellow-green) light is focused in different planes in the eye. The eye tends to focus on yellow light, so that the blue light image on the retina is not sharp.

(N.B. This may be the reason that some binocular producers realize a sharp drop in the blue spectral region (around 450 nm) of the light transmission spectra of their binoculars. The absorption of blue light below 475 nm by the rods is also rather low compared with its absorption maximum at 505-510 and the image formed from the optical signal from the rods is not sharp because of the lower resolution due to the clustering of the rods resulting in a larger ”pixel size”. That may be an additional reason for the blue drop some transmission spectra).

The rods and cones are distributed among different parts of the retina. The cones are concentrated towards the centre of the retina. In the center itself, however, there are no blue-sensitive B-cones. Just about the center of the retina is a small depression 2,5 to 3 mm in diameter known as the yellow spot (macula). The macula is composed of more than twice as many cones as rods. There is a tiny rod-free region of about 0,3 mm diameter at the center of the macula called the fovea centralis. Here the cones are thinner with diameters of 1,5 to 2 micrometers and more densely packed than anywhere else in the retina. The center of the fovea centralis does not contain blue absorbing B-cones. Cones in the fovea centralis are individually connected to nerve fibers. As a consequence the fovea provides the sharpest and most detailed information and is responsible for optimal sharp images of the eye. Simply stated the “pixel”size of the cones of the fovea is 1,5-2 micrometers. Elsewhere on the retina the situation is different. Clusters of only rods or rods and cones amounting to 200-600 rods and cones or rods only are connected with a single nerve fiber and the fiber can be activated by any of these photoreceptors. If we take the size of a rod as 6 micrometers, than an array of 10x10 rods has



Spectra of rods (grey) and cones (colored spectra)
blue = blue sensitive cones
green = green senistive cones
yellow/red = red sensitive cones

Figure 14
Impression of the spectral sensitivity of the three types of cones and of the rods in the human eye.

the size of 60x60 micrometers and is it clear that a 60x60 micrometer “pixel” size on the retina is much larger than a “pixel” size of 1,5x1,5 or 2x2 micrometers in the fovea centralis. This illustrates that the optimum resolving power of the eye is located in the fovea centralis. In the various publications different sizes of the photoreceptor clusters outside the fovea centralis are reported ranging from 100-600 photoreceptor cells per cluster. The resolution of the eye using these larger photoreceptor clusters is of course considerably lower than the resolution of single cones in the fovea centralis because of the much larger “pixel size” of the large photoreceptor clusters outside the fovea.

The rods and cones in the retina are not connected directly to the optic nerve, but interestingly enough are connected to a number of other cells that are connected to each other. Many of these cells are interconnected, while only a few carry information to the optic nerve. Through these interconnections a certain amount of information is combined from several visual receptors and “digested” in the retina. In this way the light signal is interpreted before it goes to the optic nerve and then to the visual cortex of the brain.

In the human eye the blurring due to chromatic aberration and that due to diffraction are of equal magnitude and their combined effect is minimal when the diameter of the pupil is about 3 mm. A point like object, when focused as sharply as possible, then produces a round spot on the retina with a diameter of approximately 5 micrometers. Geometrically this corresponds to an angular resolution of 1 minute of arc i.e. to the angle corresponding to an object of 30 cm seen at a distance of 1000 m or of a spot of 1 mm seen at a distance of 3,4 meter. This is the best resolution

Figure 15
Visual resolution (acuity) over the retina of the human eye.

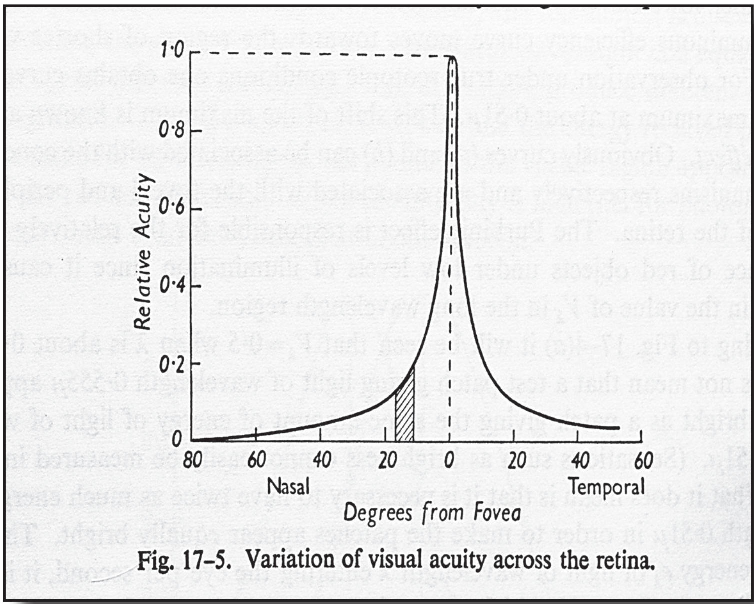
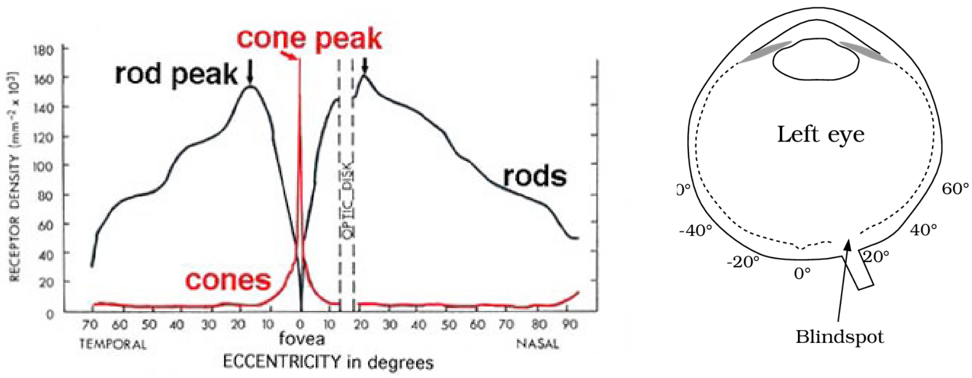


Fig. 17–5. Variation of visual acuity across the retina.

that the eye is capable of under optimal conditions. If two dots less than 1 mm apart are viewed from a distance of 3,4 m their images at the retina will fuse and they will be recorded as one dot.

With a pupil diameter of 5 mm the blurring due to diffraction is small, but the fuzziness due to chromatic aberration is larger than 1 minute of arc. Conversely, with a pupil diameter of 1 mm the fuzziness due to chromatic aberration is negligible, but the fuzziness due to refraction is more than 2 minutes of arc. In the fovea centralis the cones have a diameter of 1,5-2 micrometers, which corresponds to an angle of 1/3 minute of arc or 10cm/1000m. Thus the “pixel” size of the fovea is just fine

Figure 16
Distribution of rods and cones in a section across the eye



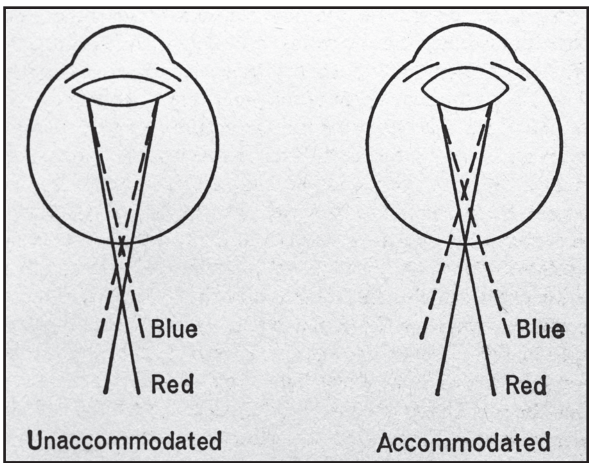


Figure 17
Chromatic aberration of the eye

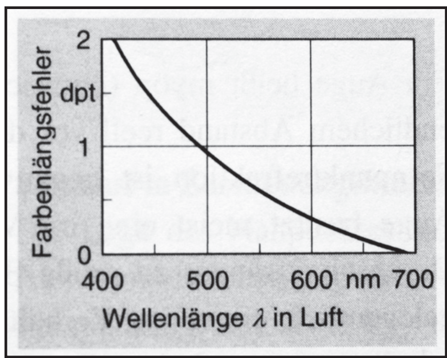


Figure 18
Chromatic aberration of the human eye as a function of the wavelength

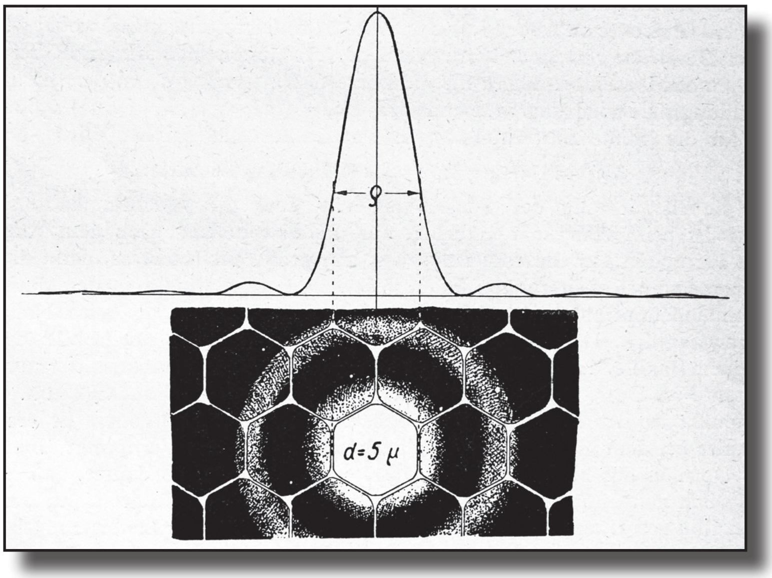


Figure 19
Projection of the main peak of an Airy disk on a cone in the fovea centralis of the human eye.

enough not to limit the visual resolving power. From the data described above it will be clear that the structural basis for optimal resolution e.g. sharp images is located in the cone vision of the fovea centralis.

If we take 1 minute of arc (=60 arcseconds) as the optimal resolution of the eye than resolution of binoculars and telescopes can be estimated as:

Telescope/binocular resolution (Res) = 60 arcseconds/binocular magnification (M) or shorter Res=60/M arcseconds.

N.B. 1 degree= 60 arcminutes ; 1 arcminute= 60 arcseconds.

4.The perception of color

For the average observer, a visual sensation is produced by electromagnetic waves whose wavelengths lie between about 400 nm and 750 nm. Different wavelengths produce different sensations of color.

For a normal eye the hues corresponding to the visual spectrum are approximately as follows:

- 400-450 nm = violet
- 450-480 nm = blue
- 480-510 nm = blue-green
- 510-550 nm = green
- 550-570 nm = yellow-green
- 570-590 nm = yellow
- 590-630 nm = orange
- > 630 nm = red.

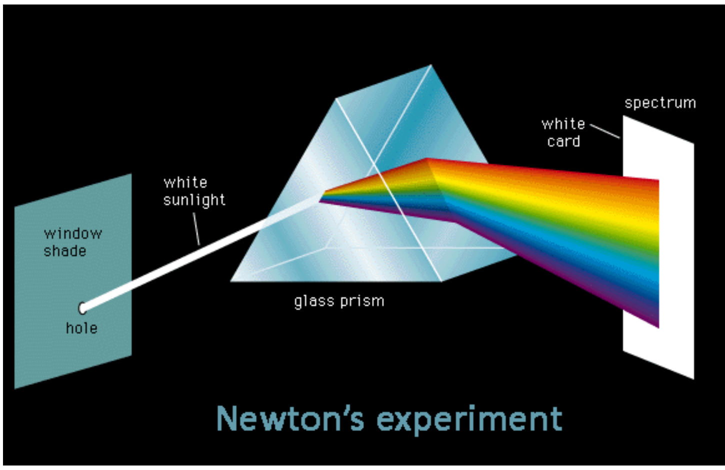


Figure 20

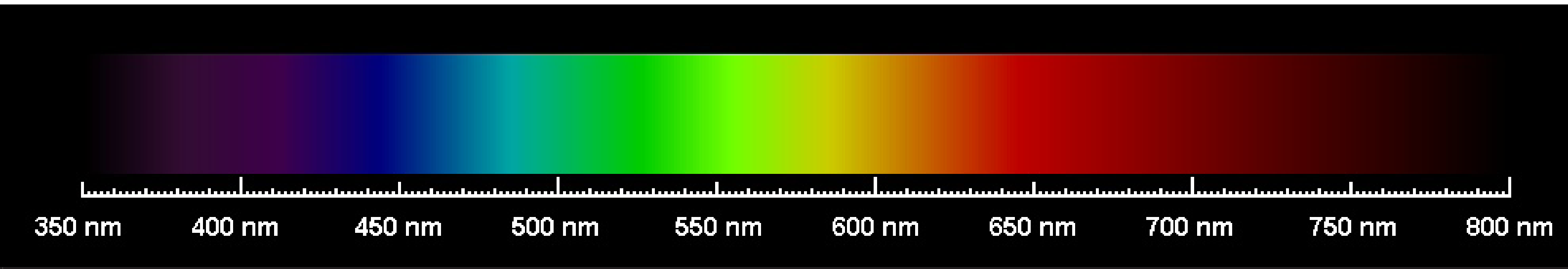
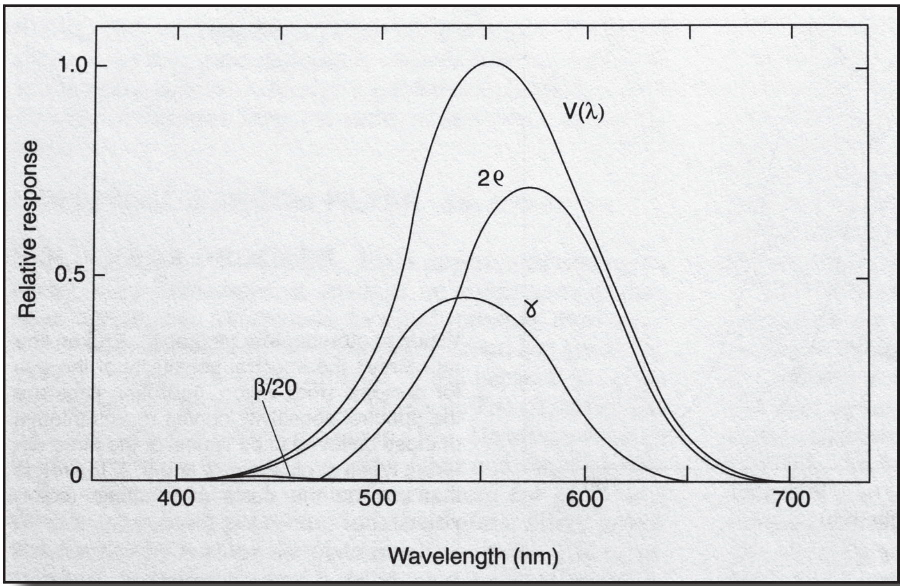


Figure 21
The daylight sensitivity curve V(λ). The contributions of the three different types of cones to this sensitivity curve are also shown, it being assumed that there are twice as many red sensitive cones (ρ) than green sensitive cones (γ) and only one-twentieth as many blue sensitive cones (β).



Color is actually a physiological sensation of the brain in response to the light excitation of cone receptors in the retina. In the human eye, the cones are sensitive to light with wavelengths between about 400 to 700 nm and there are research reports which show that the eye is even sensitive to shorter and longer wavelengths.

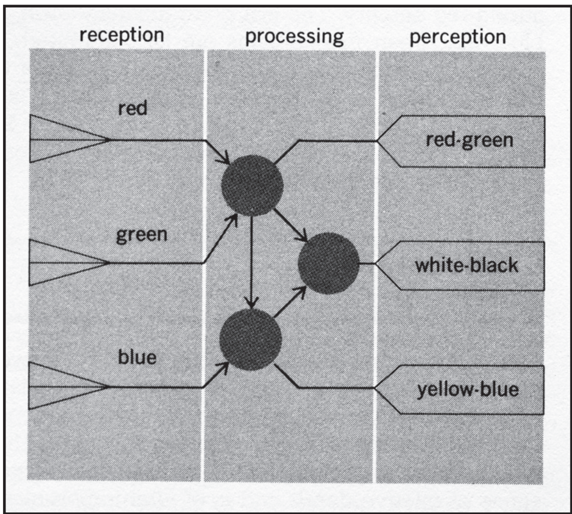


Figure 22
Schematic representation of the three phases of color vision which can be generally used to describe the various theories of color vision.

The observation and identification mechanism of these colors separately indicated as color vision is not very well understood, since it is a rather complicated process which we slowly start to understand on the basis of scientific research of the past two centuries. Many now famous names are connected with this research and I will not name them here, but the text below is based on their work.

Absorption of light in the three types of cones generates electrical signals and these signals are not directed directly to the brain, but they are transmitted via so-called ganglion cells and bipolar cells and the optical nerve to the visual cortex in the brain. The photoreceptor cells in the retina with their high amplification have a noise level which is not negligible and simply stated the ganglion cells can be said to be to check several photoreceptor reports against one another and only the "reliable" reports are transmitted to the brain. In principle the absorption of one single photon in a rod cell is sufficient for the triggering of an impulse from it. That does not mean that a single photon leads to a visual sensation. For this to occur it is, in fact, necessary that about six photons are absorbed almost simultaneously within the same small part of the retina, comprising about 500 rods. In another research report it is stated that this may even occur with two photons simultaneously. Signals which are more spread out in time or space are filtered away by the ganglion cells in the retina.

For color vision to occur absorption of light in the three types of cones generate as already mentioned electrical potentials and these electrical signals are than coded into an achromatic and two color difference signals for transmission through

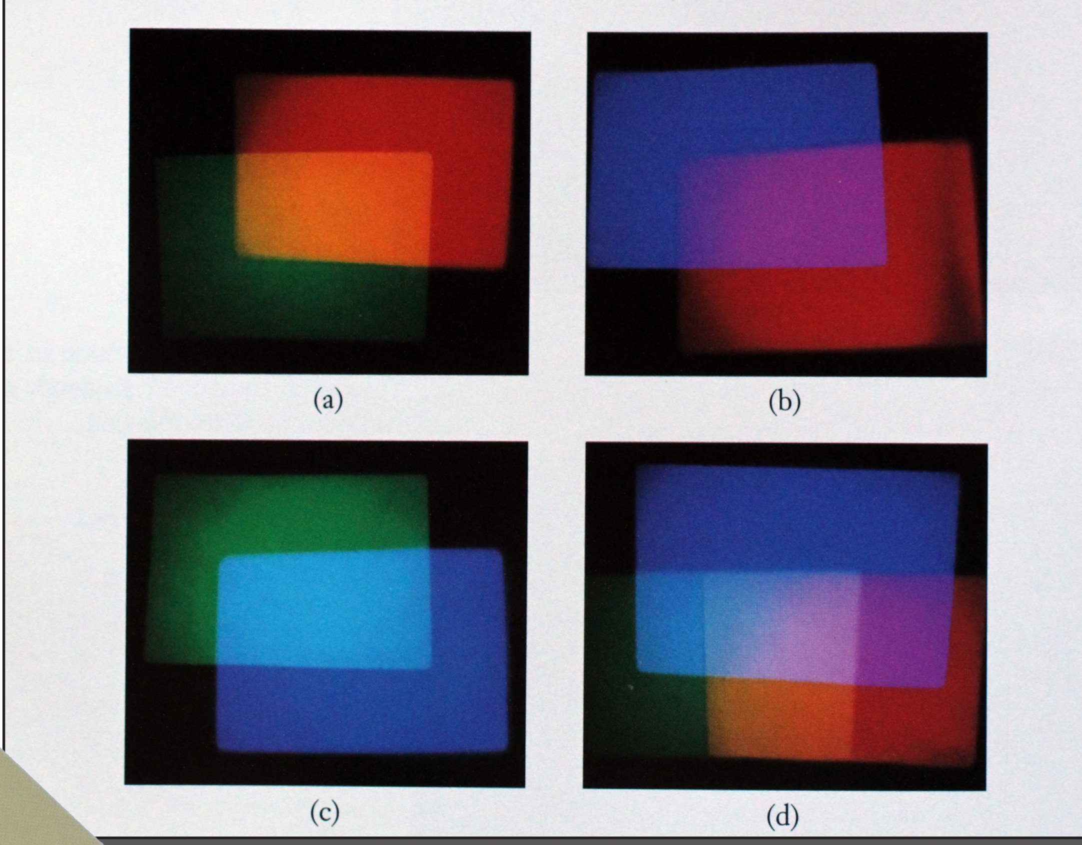
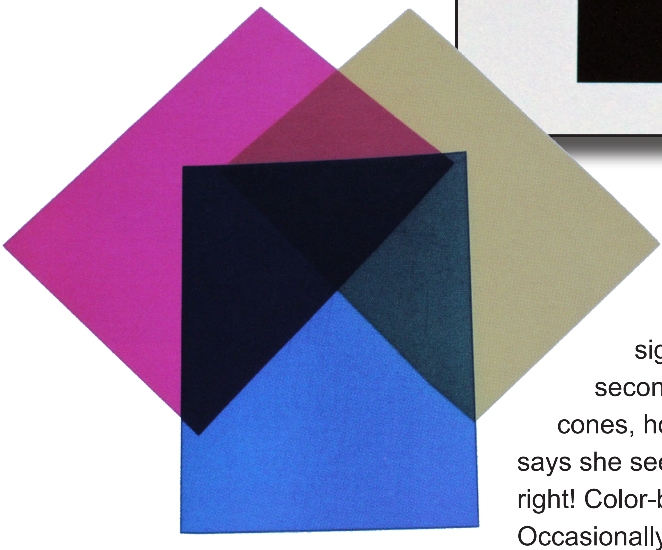


Figure 23
Additive color mixing of light: (a) red + green = yellow
(b) red + blue = magenta
(c) blue + green = cyan
(d) red + green + blue = white.

Figure 24 (below)
Subtractive color mixing by color filters:
magenta + yellow = red
yellow + cyan = green
cyan + magenta = blue
magenta + yellow + cyan = black.



the optic nerve to the brain. Although in the early stages the signals are analog voltages, when they reach the ganglion cells they are transformed into a series of nerve impulses, all of the same voltage, and the strength of the signal is then represented by the number of nerve impulses per second. Females have a slightly lower threshold of firing for the cones, however, and can see a bit more color than males. So if she says she sees colored stars and he says she does not, she is probably right! Color-blindness results when one of the primary cones is lacking. Occasionally this occurs because of failure to inherit the appropriate gene for the cone formation. The color genes are found in the female sex chromosomes, so almost all color-blind people are male (about 4 % of the male population).

The achromatic signal is produced by nonopponent cells summing the voltages produced by the B,G and R cones in the retina. This signal provides the basis for the perception of brightness, which is common to all colors. Because of the greater

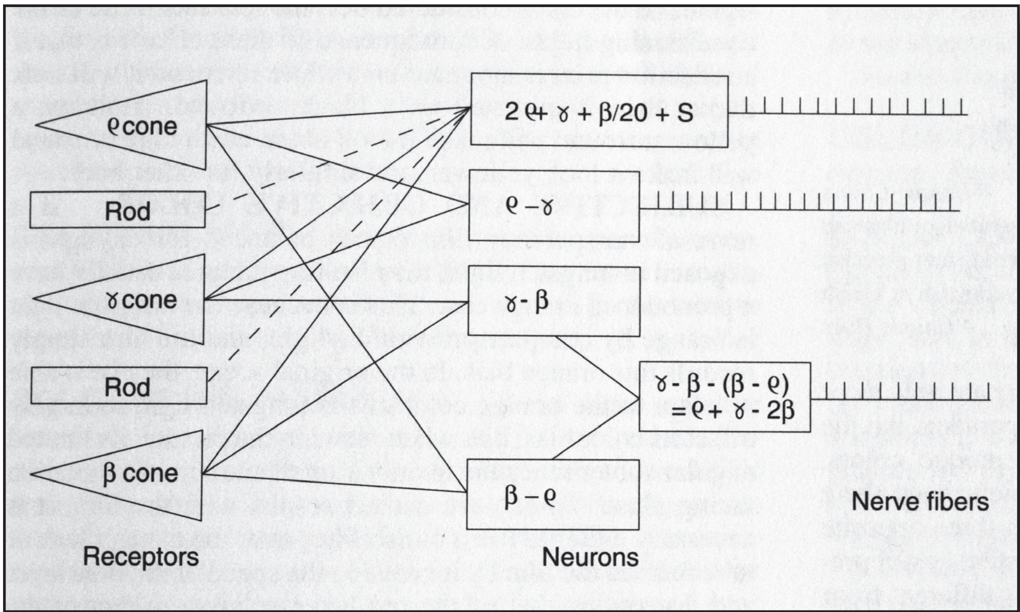


Figure 25
The visual color signals. Greatly oversimplified and hypothetical diagrammatic representation of possible types of connection between some retinal receptors and some nerve fibers.

number of R cones and the very much smaller number of B cones, the cone part of the achromatic signal is probably approximately similar to 2R+G+(1/20)B, where R, G and B are the magnitudes of the voltages produced by the three types of cones.

The rod signals are probably also transmitted along the achromatic channel to the brain, so that the complete achromatic signal is represented by (2R+G+1/20B + S), where S represents the rod signals.

In daylight, the rods play little or no part so for color reproduction at these light levels only cones are considered. The spectral sensitivity of the achromatic channel is then given by adding to the spectral sensitivity of the G cones twice that of the R cones and one twentieth of that of the blue cones. Mathematically that is not a perfect approximation due to non-linearities in the receptor response mechanism but it is a useful approximation. The resultant composite spectral sensitivity curve of the different photoreceptors can be taken as the spectral luminous efficiency function of the eye or the spectral sensitivity curve of the eye at daylight as we find that reproduced in many publications. An interesting light level is the intermediate light level between photopic (daylight) vision and scotopic (night) vision in which both rods and cones can function as light receptors. This light level of vision is called mesopic vision and to my knowledge no spectral sensitivity curve of mesopic vision is exactly known. One can expect however, that the combined spectral sensitivity of the eye for photopic and scotopic vision is addressed with mesopic vision. The mesopic vision light level is found under many circumstances during field observations and for optimal visual resolution at these light levels high light transmission of binoculars and telescopes is important.

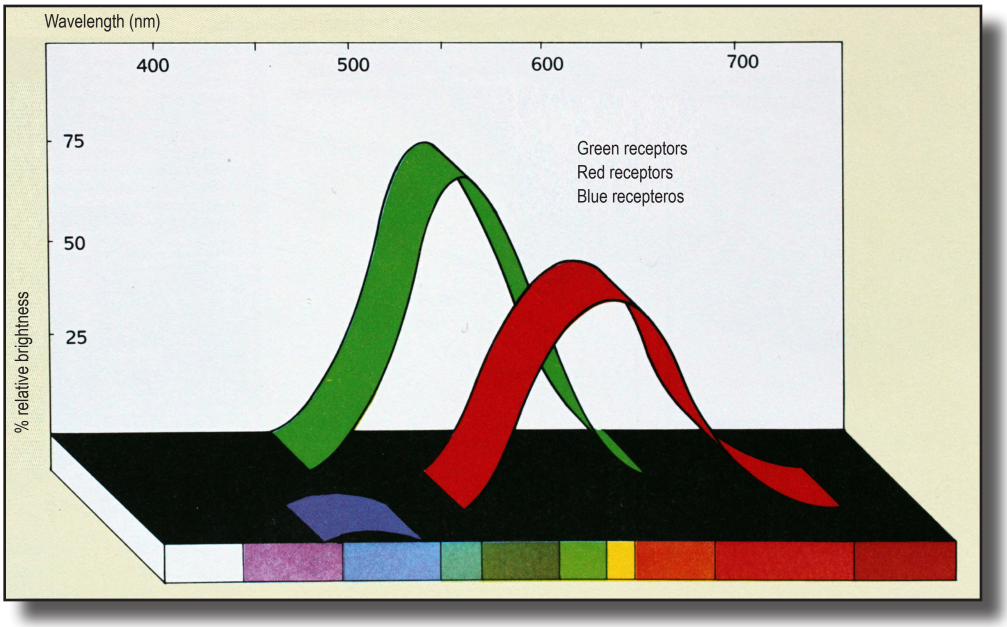


Figure 26
Spectra of the different cones and their relative sensitivity (brightness)

Light of wavelength 500 nm results in about half as much Red cone response as Green cone response, whereas light of 600 nm results in more than twice as much Red cone response as Green cone response. It is the differences in the ratios of R, G and B responses that enable differences in spectral composition to be detected independently of the amount of light present, thus providing a basis for color vision.

5. Color vision and colorimetry

Color is a visual sensation that enables us to distinguish otherwise identical objects. Color is generally associated with light of a certain wavelength or combination of wavelengths, but equating color to wavelength can be misleading, because a given wavelength may appear to have different color under different conditions, and we may see a certain color even when the wavelengths generally associated with it are absent. Most colors are due to a distribution of wavelengths. Thus one way to describe a certain color might be to show an intensity distribution function. The transmission spectrum of an optical system is such an intensity distribution function (a graph of intensity versus wavelength). If the curve has a prominent peak in the green portion of the visible spectrum, the observer most likely perceives this light as having a green color. When the intensity distribution function has several peaks, it is not so easy to predict the color that will be perceived. In practice the eye cannot extract all the information in an intensity distribution curve by looking at the light. Indeed, many lights with different curves appear to have the same color. Rather than specifying the intensity distribution, then, it is more profitable to specify three qualities of colored light that determine how the light appears: hue, saturation and brightness.

- **HUE** refers to the color name or what distinguishes one color from another; all reds differ in their hue from all blues regardless of other qualities. Hue s specified as a dominant wavelength in the visible spectrum that best matches the perceived color, even though light of that particular wavelength may not be present.
- **SATURATION** corresponds to the “purity” of the color; a saturated color has its intensity concentrated near the dominant wavelength, while an unsaturated color includes contributions from other wavelengths as well. Spectral colors have the greatest saturation; white light, which consists of all colors, is unsaturated.
- **BRIGHTNESS (OR LIGHTNESS)** refers to the sensation of overall intensity, ranging from dark, through dim, to bright. Other terms used to describe brightness are value, brilliance and luminosity.

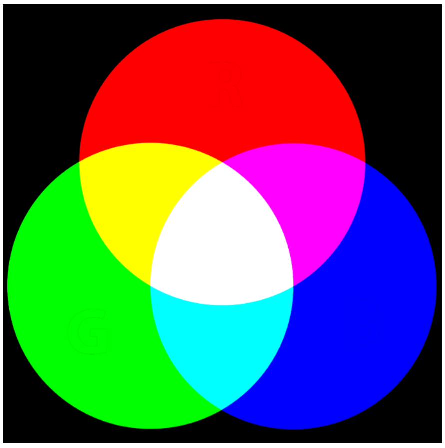


Figure 27
Combination of blue, green en red yield white light

It is possible to mix red, green, and blue light in the right proportions to obtain nearly any desired color. We can even obtain white light.

So, white light is a mixture of all visible frequencies (wavelength) or colors, although perhaps not of equal intensities. The composite colors can be easily demonstrated by dispersion of white light by a prism or by dispersion of sunlight at water droplets in the air, which generates rainbows. When colors of light are mixed together, it is found that one does not need all colors to make white light. Just the combination of red, green and blue light will do the job. When light beams of these colors are projected on a screen so the colored beams overlap, additive mixtures of colors and white are produced. Evidently the excitation of all types of cones in the eye causes the combination of signals to be interpreted as white and other combinations of different colors. This is referred to as the additive method of color production. By adding various amounts of red, green and blue light, we find that a wide variety of colors and hues of the visible spectrum can be generated. As a result, red, green and blue are called additive primaries or primary colors. Not only does a mixture of the additive primaries appear white to the eye, but many pairs of color combinations do also. The colors of such a pair are said to be complementary colors. The complement of a primary color is called a secondary color. For example the complement of blue is yellow, of red is cyan and of green is magenta. This is not surprising since a combination of red and green is interpreted by the brain as yellow. Presumably, yellow light stimulates the red and green cones, which along with blue cone stimulation is a “white” combination. Thus we see that an object has color because of the light coming from light sources or by reflection from surfaces. When white light strikes transparent red glass or a red rose, only red light is transmitted through the glass or reflected from the rose. All of the other colors are absorbed.

How many colors can we see?

Most observers can discriminate about 150 steps between 380 nm (violet) and 700 nm (red). However, we can obtain many more steps by changing the saturation and brightness as well.

Afterimages

If you stare at one color for some time and then quickly shift your attention to a white sheet of paper, you will generally see a negative afterimage in a complementary color. This is a type of adaptation; after a period of stimulation, the receptors become less sensitive to further stimulation and are thus fatigued. If you stare at a red object, for example, the red receptors become fatigued, but when you shift to a white object the blue and green receptors are fully stimulated and the paper appears blue-green. Even after a color stimulus is removed from the retina, the receptors continue to be active for a short time, and this leads to a positive afterimage. To observe a positive afterimage, close your eyes for several minutes and cover them with your hands so that the residual effects of previous light have died away. Open your eyes for a second or two and stare at some brightly lighted patch of color. Then cloes your eyes again, and you should see an afterimage of the object with its original color.

Contrast

Contrast may increase when colors are placed side by side. If orange is presented next to yellow, for example, the orange appears redder and the yellow greener. Two colors that appear to be identical when viewed separately may appear quite different when viewed side by side. Simultaneous color contrst suggest that there is chromatic lateral inhibition. Helmholz explained simultaneous contrasts as due to afterimages; ordinarily you do not stare at a single spot but rather roam the visual field, and you thus create weak overlapping afterimages of which you are unaware. If you are presented with two adjacent areas of different color, your eyes will flick back and forth between those two areas, and each color you see will be a combination of the true color of that area and the afterimage of the adjacent area.

In addition to simultaneous contrast, there is also a successive contrast effect. If you look for a period of time at an area of one color and then look quickly at an area of a different color, the color of the second area will be modified. For example, if you stare at an orange piece of paper for a while and then look at a green piece of paper, the green paper will take on a definite blue-green appearance for a period of time.

Color constancy

Objects tend to remain the same perceived color even though the coloration of the illumination may change. The light form the blue sky is quite different from the light of a bonfire, yet the same object seen under these two different illuminations may appear to have essentially the same color. The so-called color constancy apparently also involves chromatic lateral inhibition. For example, an overall excess in red illumination is ignored because the increased stimulation at the center of the receptive field and inhibition in its surroundings cancel each other. There is a limit to color constancy, of course. If we view a blue object in illumination that has no blue light whatsoever (such as the yellow light form a sodium lamp), it will appear black. However, it is surprising how very little blue light needs to be present in order for us to identify it as being blue. The remarkable ability to adapt to the color quality of the prevailing illumination is sometimes called general color adaptation.

Color systems

Although no color dictionary can describe the full range of distinguishable colors, several attempts have been made to systematically specify colors. Colors may be arranged, according to hue, saturation and lightness, in a color tree, see for example the color trees devised by Albert H. Munsell and by Ostwald.

A chromaticity diagram, developed by the CIE (Conseille International d'Eclairage or International Commission of Illumination) makes it possible to describe color samples mathematically and to represent the dominant wavelength and purity of the sample on a diagram. To develop the CIE system, data were needed on the color-matching characteristics of the eye. Using a colorimeter, a number of observers matched colors against a spectrum of primary colors. Based on the results, three color-mixture curves were produced; these represent the relative amount of each of the primaries needed by the average observer to match any part of the spectrum. These curves are labeled z, y and x, and they correspond to the “red”, “green”, and “blue” primaries. The y curve is identical to the sensitivity curve of the human eye, so multiplying the y curve times the spectrum to be analyzed gives the brightness of the spectrum. Corresponding to the color matching functions are three tristimulus values of the spectrum $\overline{X}$, $\overline{Y}$ and $\overline{Z}$. These are found by multiplying the x, y and z curves times the spectrum in question. For example, gives

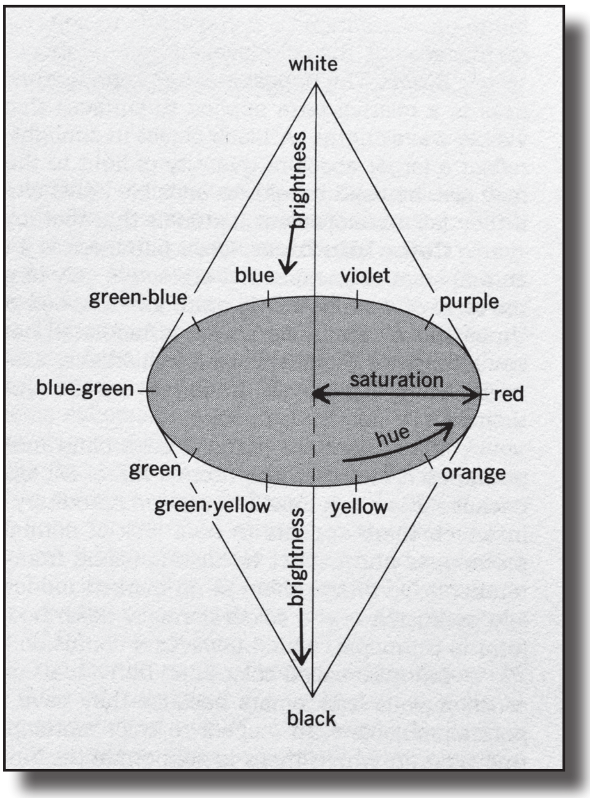


Figure28
Diagram of a simple color tree to illustrate the appearance of surface colors.

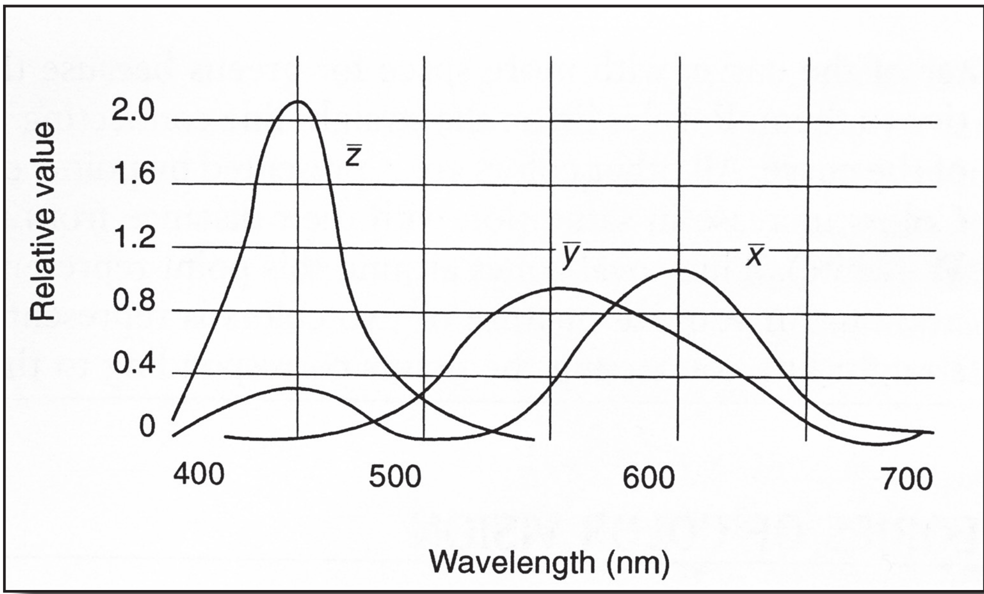


Figure 29
CIE color-matching functions

$\overline{X}$ the amount of "red" primary needed to match the spectrum being analyzed. To construct a chromaticity diagram, the color coordinates x, y and z are defined as follows:

$$x = \overline{X} / \overline{X} + \overline{Y} + \overline{Z} \quad y = \overline{Y} / \overline{X} + \overline{Y} + \overline{Z} \quad z = \overline{Z} / \overline{X} + \overline{Y} + \overline{Z}$$

From the definitions, it is clear that x+y+z=1. This means that if two of the coordinates are given, the third one is automatically determined. It is customary to use the coordinates x and y in constructing chromaticity diagrams as shown in the added figure. The CIE chromaticity diagram or tristimulus diagram shows colors determined by the CIE graphs and equations. The CIE system works on formulas,

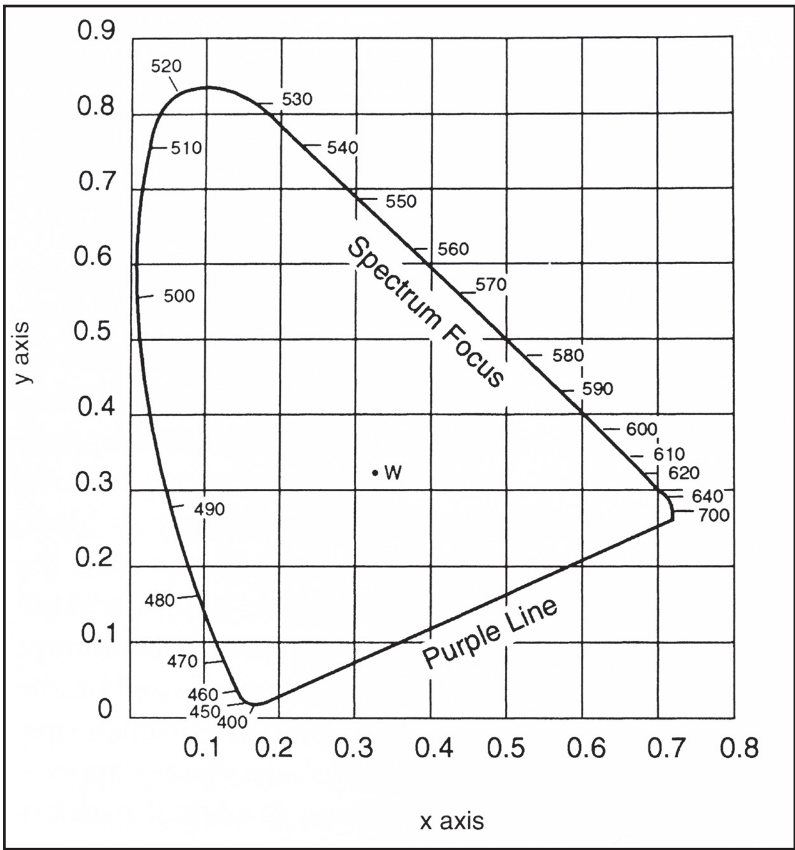


Figure 30
A CIE-type chromaticity diagram. The entire spectrum of fully saturated hues (from 400 to 700 nm) lies on the smooth outer curve. White lies near the center at point W.

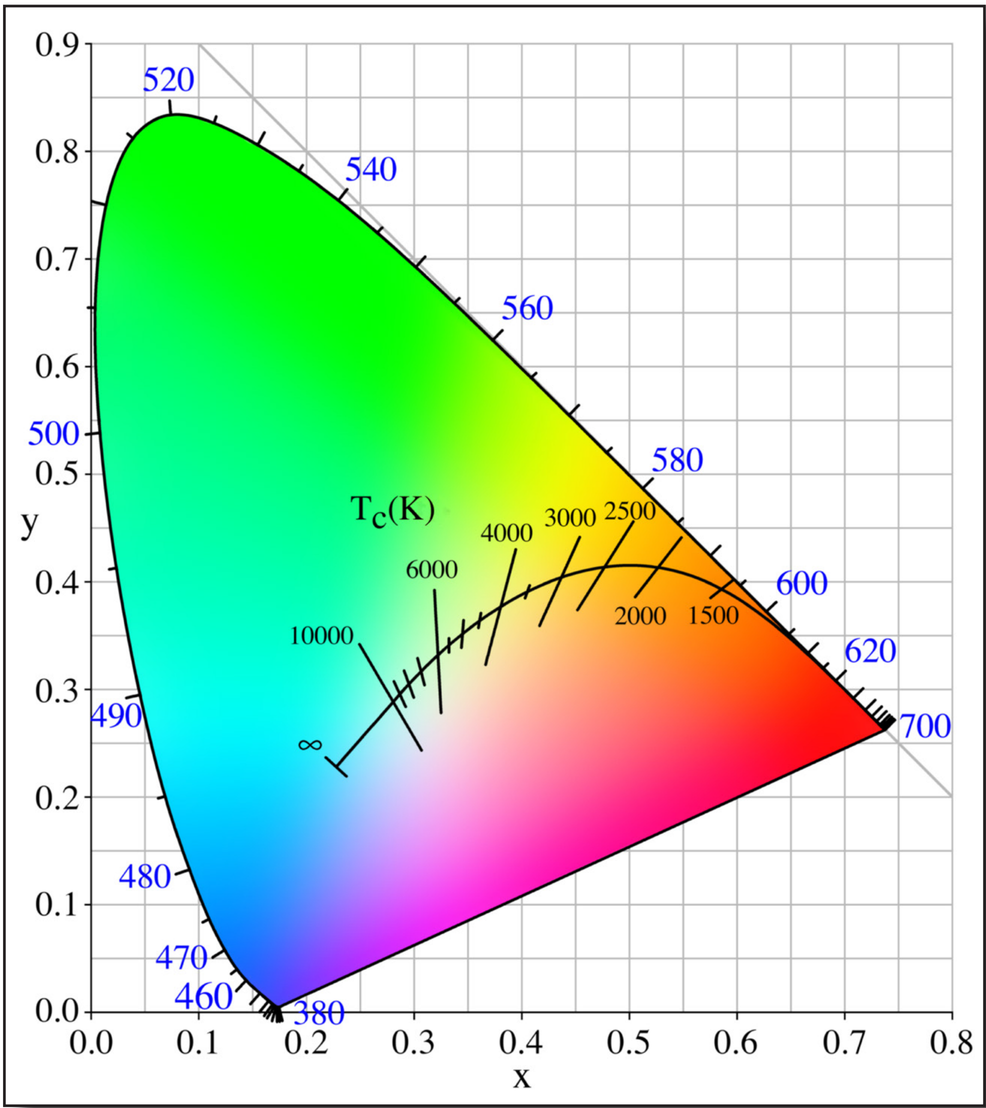


Figure 31
CIE chromaticity curve in color, see also figure 30

not on color samples. The entire spectrum of fully saturated hues (from 400-700 nm) lies on the smooth outer curve of the CIE diagram, with more space for greens because the eye is more sensitive to them. Purples lie on the straight line connecting the red and blue ends of the curve. All other colors are represented by pints enclosed by the curve. Colors increase in saturation with their distance from the achromatic point W (white). Thus oval zones around this point represent colors of equal saturation. An additive mixture of the two colors is represented by a point on the straight line connecting the points corresponding to these two colors.

6. Brightness, luminance and color

Brightness can be defined as the visual experience that is approximately correlated with the luminance of objects seen as light sources. Since brightness is a psychological concept, there are no units of measurement as there are for luminance, a psychophysical concept. Luminance is the property of a source or surface that most closely correlates with the subjective perception of brightness, which, because of adaptation and contrast effects of the visual system, cannot be used as a reliable measure. A confusing array of units has been used to specify luminance. The preferred International System luminance unit is the candela per square meter. This unit is derived in terms of intensity per unit projected area.

However, brightness is not only determined by the light intensity the eye is exposed to. The colour hue also plays a role in our perception of brightness. Our brain judges yellow for example as a bright color, whereas that is not so for violet. If we judge the image quality of a binocular or telescope as being very bright than there the light intensity offered by the instrument to the eye is of great importance and that is determined by its exit pupil in combination with its light transmission level. In addition to this light level the perception of image brightness is also influenced by the color balance of the light transmitted by the instruments.

7. Resolution and resolution limits

Resolution is defined as the capability of distinguishing between two separate elements or details in an image. In paragraph 3 the visual resolution and the factors that govern this process were already described and the outcome was that the optimum resolution of the eye is approximately 1 minute of arc at high light intensity (daylight) and it drops to 4-13 arcminutes in twilight). In the box below I have repeated the text that handles the visual resolution:

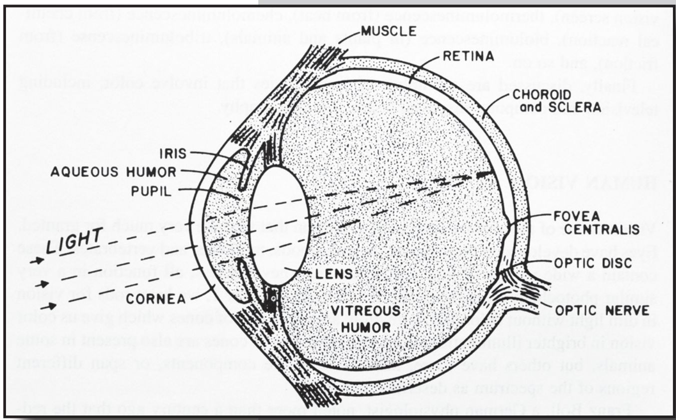


Figure 32
The human eye

Resolution of the eye:

In the human eye the blurring due to chromatic aberration and that due to diffraction are of equal magnitude and their combined effect is minimal when the diameter of the pupil is about 3 mm. A point like object, when focused as sharply as possible, then produces a round spot on the retina with a diameter of approximately 5 micrometers. Geometrically this corresponds to an angular resolution of 1 minute of arc(= 60 arcseconds) i.e. to the angle corresponding to an object of 30 cm seen at a distance of 1000 m or of a spot of 1 mm seen at a distance of 3,4 meter. This is the best resolution that the eye is capable of under optimal conditions in daylight. If two dots less than 1 mm apart are viewed from a distance of 3,4 m their images at the retina will fuse and they will be recorded as one dot.

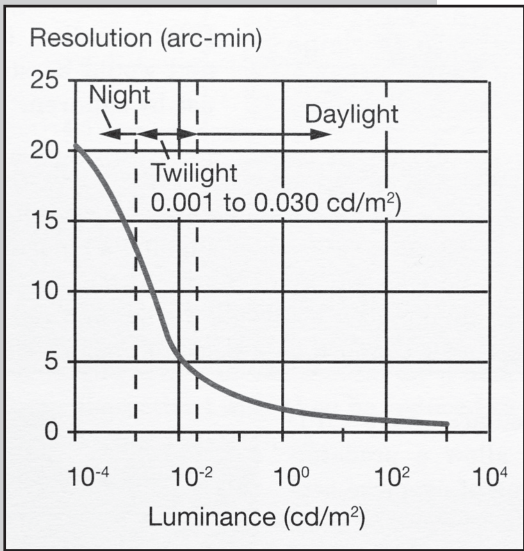


Figure 33
Visual resolution as a function of light intensity.

With a pupil diameter of 5 mm the blurring due to diffraction is small, but the fuzziness due to chromatic aberration is larger than 1 minute of arc. Conversely, with a pupil diameter of 1 mm the fuzziness due to chromatic aberration is negligible, but the fuzziness due to refraction is more than 2 minutes of arc. In the fovea centralis the cones have a diameter of 1,5-2 micrometers, which corresponds to an angle of 1/3 minute of arc or 10cm/1000m. Thus the "pixel" size of the fovea is just fine enough not to limit the visual resolving power. From the data described it will be clear that the structural basis for optimal sharp images is located in the cone vision of the fovea centralis. This is the situation for daylight vision, but the situation changes dramatically for twilight and night vision. In these lower light levels the eye at first transfers to mesopic vision in which both rods *and cones function as light receptors. Since they are organized in larger clusters the eye's resolution decreases to about 4-13 arc minutes in twilight and 13-30 arc minutes at night (cone vision only). These numbers can vary from one individual to another and the variations can be in the order of +/- 20%. From these data the optimum visual resolution A of a telescope or binocular with magnification M is than estimated as A=60/M arcseconds. Below we will treat the theoretical boundaries of binocular/telescope resolution and the factors that influence it.*

Figure 34
George Bidell Airy (1801-1892). Astronomer and mathematician in Greenwich.

To understand how the resolving power of a binocular or telescopes comes about we have to take into account the processes that occur when light waves interact with glass. The result of it is that the resolving power of a binocular or telescope depends on the size of the diffraction pattern of the transmitted light and the distribution of the intensity in it. In 1835 Sir George Bidell Airy (1801-1892) succeeded in making the mathematical analysis of the diffraction pattern of a circular aperture as found in telescopes and binoculars and he calculated the exact values for the diameters and intensity distribution in the diffraction pattern. In the center there is an intensity maximum. This is separated by a dark ring from the first diffraction ring, and so on. According to Airy's calculations an ideal system with a circular aperture the central disk contains 84% of the total light from a point source. The first diffraction ring contains 7%, the second 3% and so on. Because of this intensity distribution our eyes generally "observe" only light of the central disk, since the intensity of the other diffraction rings is too low to be detected by the eye.

The diameter of the first dark ring is 2,44 x λ/D radians, where λ is the wavelength of the light and D is the aperture of the binocular or telescope (= its objective diameter). Because one radian is 206,265 arc seconds, for green light (555 nm) the angular diameter of the first dark ring is 280/D arcseconds when D is in millimeters. For red light (656 nm) the angular diameter is 330/D arcseconds and for violet light (404 nm) 203/D arcseconds. For small instruments Dawes empirically determined the resolving power as 117/D arcseconds.

N.B. For the attentive reader it will be clear that the theoretical resolution of a binocular or telescope is only dependent on the size of its objective diameter. However, we will illustrate below that there is a range of factors, which affect the values of this theoretical resolution.

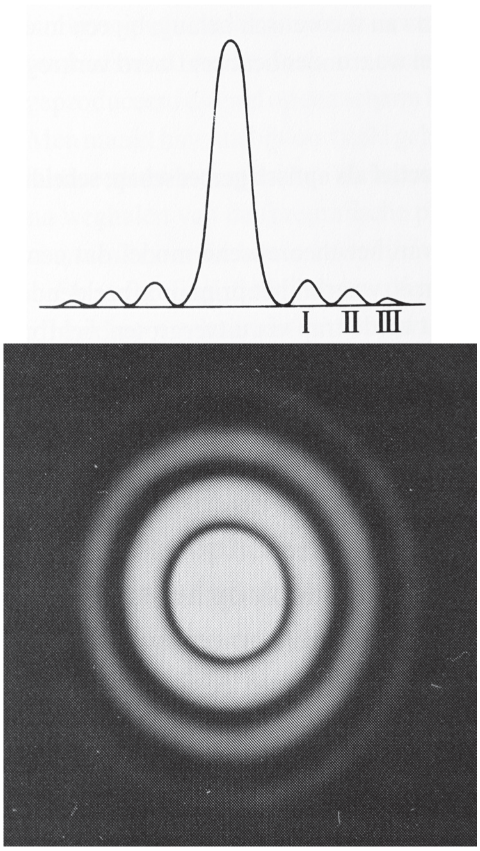


Figure 35
Diffraction pattern of a circular aperture: airy disk (below) and its projection (top).

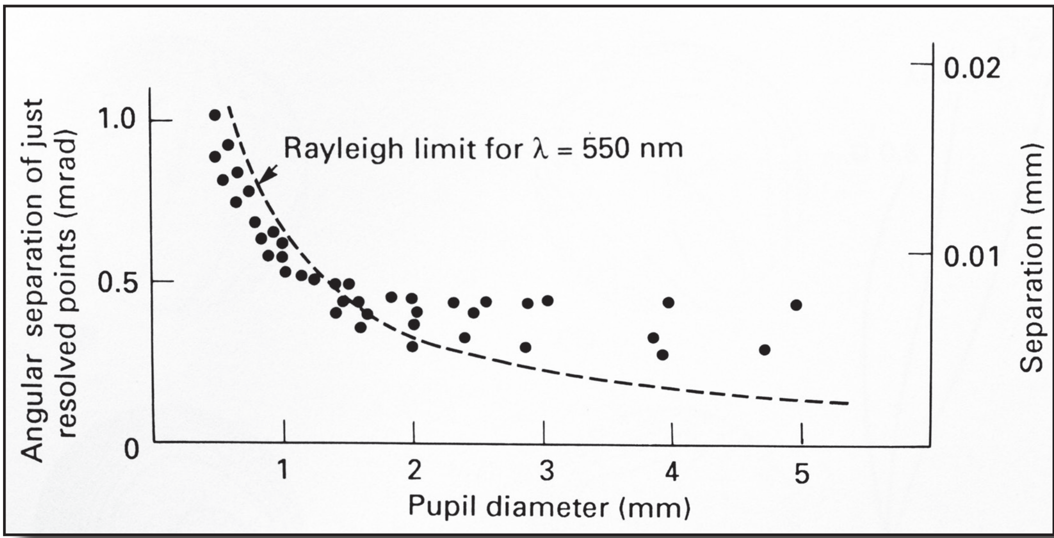


Figure 36
Diffraction and the resolving power of the human eye.

From the size of the Airy disk we might expect that a binocular could easily distinguish two Airy disks separated by their own angular diameter. Rayleigh (J.W. Strutt, later Lord Rayleigh 1842-1919) showed that stars with intensity maxima separated by roughly half that distance can be distinguished visually. The diffractions patterns are than separated by 140/D arcseconds. The image seen by the eye is the sum of light patterns. For stars of equal brightness, the intensity in the dip between the stars' maxima is 74% of the maximum. Although Rayleigh defined this angular distance as the resolving power of a telescope or binocular, he realized that even closer stars could be distinguished. For small instruments William Rutter Dawes (1799-1868) empirically determined the resolving power as 117/D arcseconds (D in mm) (or 11,58/D arcseconds when one prefers D in cm), which implies a dip of only 3,2% between the intensity maxima. As a consequence one can conclude that differences in light transmission of 3% or less are not useful for a better resolution of details with a binocular or telescope. The Rayleigh limit is defined as 1,22λ/D radians = 14,1/D seconds of arc (at 560 nm, D in centimeters) or 13,8/D seconds of arc at 550 nm.

We have to add, however, the following limitations for the use of the Dawes limit:

1. The Dawes criterion is strictly valid only for white double stars consisting of two sixth magnitude components, viewed with a 150 mm telescope.
2. The diameter of the Airy disk, thus the resolving power depends on the wavelength of the light. The Dawes criterion, for instance, does not apply to red double stars.
3. For stars of unequal brightness, the dip in the combined Airy pattern will be less favorable, so distinguishing the star images will be more difficult. Lewis found a 3 times worse resolving power for a double star with magnitude 6.2 and 9.5 and 8 times worse for a pair with magnitude 4.7 and 10.
4. The Dawes criterion is only valid when the diffraction pattern has the ideal intensity distribution function. Optical aberrations affect this distribution and decrease the resolving power of the binocular or telescope.
5. Air currents smear the combined Airy pattern of the double star. Only after careful examination under good seeing conditions can definitive conclusions with respect to the optical performance of telescopes and binoculars be drawn.

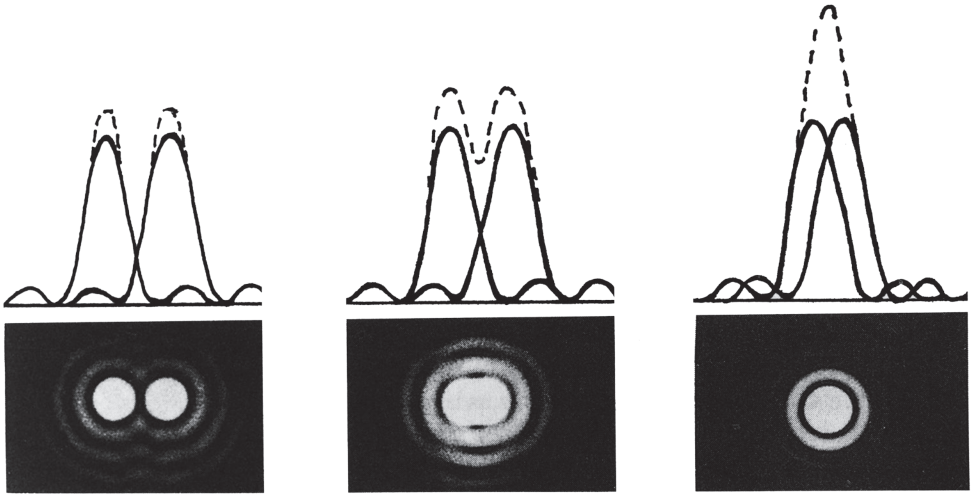


Figure 37
Diffraction patterns due to two pinholes with varying spacing. In (b) and (c) the holes are so close together that the central peaks in their diffraction patterns overlap.

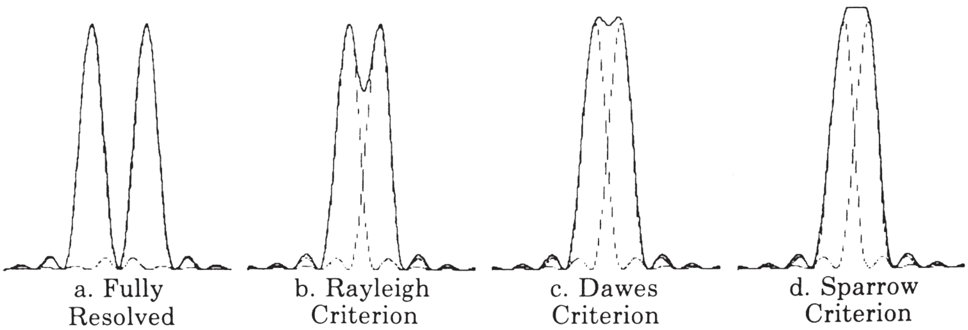


Figure 38
Resolution for equally bright double stars.

Despite the limitations of the Dawes criterion quite a few sources use it to calculate the theoretical resolution of binoculars. When one investigates the literature different expressions for the Dawes limit can be found. The Rayleigh limit is defined as dependent on the wavelength.

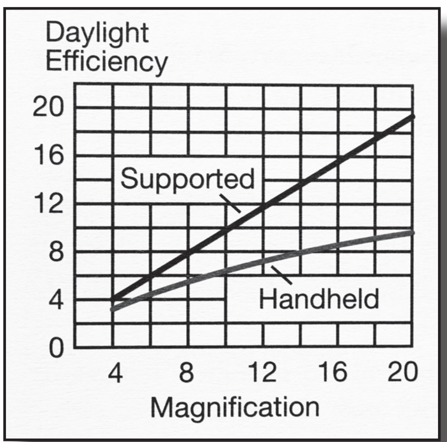


Figure 39
Visual resolution for handheld binoculars is lower than for supported binoculars and this effect increases with increasing magnification.

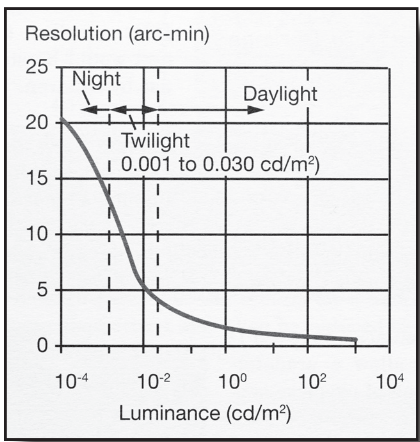


Figure 40
Visual resolution as a function of light intensity.

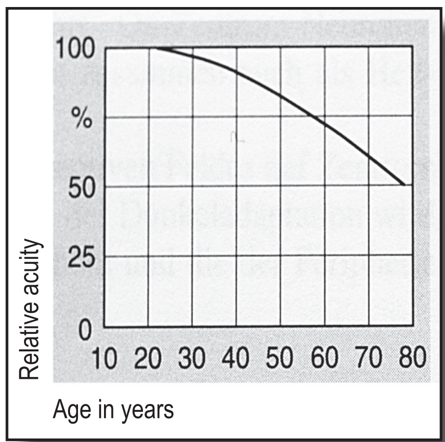


Figure 41
Visual resolution decrease with increasing age.

TABLE

Theoretical resolution of binoculars with different objective diameter and using different values of the Dawes and Rayleigh limits.

Objective lens diameter	Resolution in arcseconds using Dawes limit 116/D	Resolution in arcseconds using Dawes limit 117/D	Resolution in arcseconds using Rayleigh limit 138/D (550nm)	Resolution in arcseconds using Rayleigh limit 140/D (560 nm)
20 mm	5,8	5,85	6,9	7
24 mm	4,8	4,9	5,75	5,8
30 mm	3,86	3,9	4,6	4,7
32 mm	3,62	3,65	4,3	4,38
40 mm	2,9	2,92	3,45	3,5
42 mm	2,76	2,79	3,28	3,3
50 mm	2,32	2,34	2,76	2,8
56 mm	2,07	2,09	2,46	2,5
60 mm	1,93	1,95	2,3	2,33
80 mm	1,45	1,46	1,72	1,75

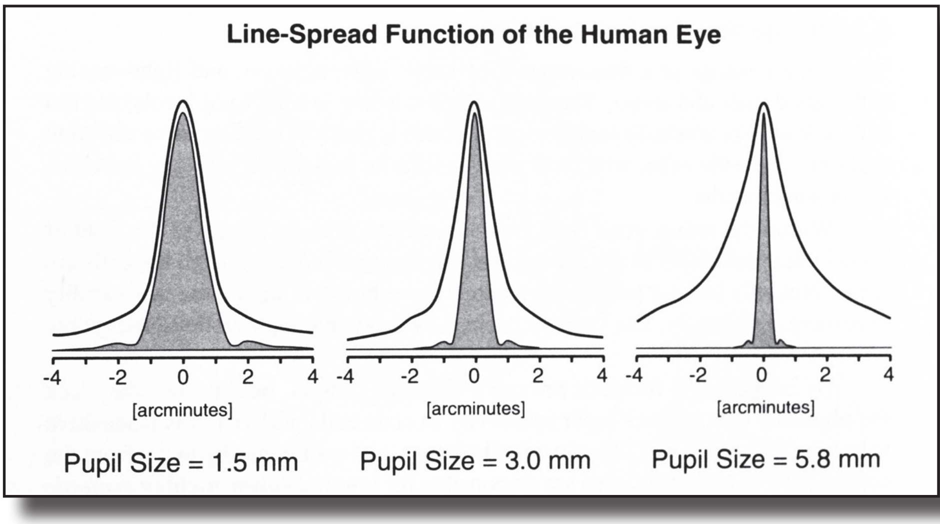
Hallock Smith et al. make useful critical remarks about resolution limits with telescopes in their chapter 2.3.3 (which also apply to binoculars). They give the following expression which relates the exit pupil of a telescope to its resolution: R= 279/d arcseconds, where d is in mm and they add the following remarks to this formula:

"Under good lightning conditions, observers with a normal vision can just distinguish a uniform disk 2 arcminutes in diameter from a point source. Using this value and solving for d, we find that when the exit pupil of the telescope is 2,3 mm, an observer with normal vision can just barely distinguish that a star image is not a perfect pinpoint of light. In other words, a telescope with an exit pupil of 2,3 mm is working at the highest magnification that produces star images barely distinguishable from perfect pinpoints. However, the criterion we adopted – "barely distinguishable"- is too demanding. With a 1,5 mm exit pupil, star images will appear ever-so-slightly larger than perfect pinpoints – still close enough that for all intents and purposes they are perfect pinpoints of light. To translate this important exit pupil diameter into magnification, we divide the aperture of the instrument, in millimeters, by 1,5 mm. For a telescope with an aperture of 200 mm, the magnification works out to 133x. If the instrument has a focal length of 2000 mm (f/10), an eyepiece with a focal length of 15 mm produces a 1,5 mm. We want o stress that no sharp boundary exists between "pinpoint star images" and "ever-so-slightly visible Airy disks". Eyepieces with focal lengths ranging from 12 to 18 mm produce the same effect: on nights of good seeing, stars will appear nearly like diamond dust scattered across the field of view. For the eye to be fully diffraction-limited, you must be able to clearly see the Airy disk even though the image will not subjectively appear as a pinpoint.

With a small- to moderate aperture telescope, an exit pupil of 0,5 to 1,0 mm is usually necessary. With a large observatory instrument, atmospheric turbulence "seeing"- overwhelms the Airy disk, and a larger exit pupil is necessary.

The iris opening giving the greatest resolution with the unaided eye lies in the range of 3 to 4 mm. It stands to reason that the subjectively sharpest telescope

Figure 42
Intensity profiles across images formed by the human eye at different pupil sizes. When the pupil is constricted in bright light (pupil size 1,5 mm), the theoretical diffraction profile (grayed-in shape) nearly matches the actual performance profile (area under thck line) of the eye. At lower light levels, the eye's aberrations increasingly dominate its imaging capability.



images will occur with the same size exit pupil. For this reason many deep-sky observers prefer to employ a 3 mm to 4 mm exit pupil. It is not clear whether these sharp images occur because light is optimally concentrated on the retina (N.B. Van Ginkel: see also the Stiles-Crawford effect and the density distribution of rods and cones over the surface of the retina), or because the image as a whole is subtly and subjectively pleasant to view".

8. Factors affecting resolution

In the previous paragraph 7 the theoretical resolution limits of our eyes and binoculars and telescopes are discussed. However, real life does not proceed via theoretical limits when using binoculars and telescopes.

The visual resolution is reduced if the observed object has inherently low contrast, as is the case for most natural objects. Reductions in contrast also occur because of lightning irregularities, atmospheric effects, diffraction, aberrations in the eye, focus errors, and effects of misalignments and aberrations in any optical system employed.

So, the visual resolution using binoculars and telescopes is affected by different factors, since the image of the natural objects is far more complex than an image of a point source. The image consists of a multitude of details having different size, shape, color, brightness, and contrast –a virtually infinite number of bright and less bright point sources. Each of these contributes a diffraction pattern to the focal plane, so the final image is the composite of the overlapping diffraction patterns.
In the case of a bright surface and an adjacent dark surface, diffracted light encroaches into the dark border, causing blurring and unsharpness of the borderline. A thin dark line on a dark background is "greyed", while a bright line on a dark background is widened. These effects are visible particularly when these lines have an angular width comparable with or smaller than the diffraction pattern. Depending on the shape, size, brightness, contrast and color of the object observed, the influence of diffraction on the final image will be different. In that case it is difficult to find a representative and reproducible method to define the resolution of an optical system for this kind of image. It was the concept of contrast transfer for optical systems developed in 1946 by P.M. Duffieux, which yielded considerable insight into what happens in the image forming process. For details to be visible they must have sufficient contrast. If the image contrast lies below the eye's visibility

threshold, then the detail will be invisible. Image contrast depends not only on the inherent contrast in the object, but also on how much contrast the optical system transfers from the object to the image plane. Contrast transfer is the key for understanding why a certain object detail may be visible in one binocular, but not in another of the same aperture.

Resolving power and contrast transfer are both quality criteria for every binocular and telescope. Today it is possible to measure the contrast transfer of an optical system with special equipment and the relation between image contrast and resolution can be determined for every point in the image plane with so-called Contrast Transfer Functions (CTF), the Modulation Transfer Function (MTF) or the Optical Transfer Function (OTF).

In principle, contrast transfer is measured by placing a grating having a sinusoidal intensity distribution as an object in front of the optical system, then measuring the contrast of the resultant image. The ratio between image contrast and object contrast is called the Contrast Transfer coefficient CT. Each combination of a bright line and a darker line is called a line pair (lp). A coarse target has a small number of line pairs per millimeter in the target grating, while a fine target has a high count of line pairs per millimeter (lp/mm). To evaluate an optical system, we vary the spacing of line pairs in the grating, and measure the contrast in the image. As the number of line pairs increases, the optical system renders then with lower and lower contrast because every point in the object is represented by a diffraction pattern in the image.

This diffraction pattern scatters light around every image point so that the dark places in the image are illuminated by diffracted light. This effect becomes more important as the distance between elements in the image approaches the size of the diffraction pattern. At some value the image contrast is reduced to zero. The image of the grating will then be uniformly bright and without any structure. This is the highest resolving power the system can attain. Duffieux found that the contrast function for a perfect system is a smoothly decreasing monotonic curve. CTF curves are extremely useful because we can compare the performance of an imperfect optical system with the curve for a perfect system. Since the CTF of real systems is the accumulation of both diffraction effects and various aberrations, we gain information about the magnitude of image aberrations. The curves for imperfect systems generally lie below the ideal CTF curve. This means that for the same resolution, the image contrast of the imperfect system is lower than that of a perfect system. Image aberrations usually lower the CTF curve more at large numbers of line pairs per millimeter than at low numbers because the diffraction rings are brightened at the cost of the Airy disks.

The CTF curve gives a better overall picture of the binoculars' or telescope's optical quality, and certainly yields far more information than testing on double stars possibly can. It takes into account not only the accumulation of diffraction effects but also the imperfections in the optical system, not only errors of fabrication but also of design. The net capability of the binocular or telescope finds its expression in the position of the contrast transfer curve with respect to the idealized curve. For visual observation of low contrast details on objects it is difficult to define a meaningful resolving power for a binocular or telescope. Parameters such as brightness of the image, intrinsic contrast, image aberrations, and magnification as well as the contrast sensitivity and visual acuity of the eye must be taken into account. Because of this, any definition of resolving power is always subject to strict conditions.

Because the resolving power for high contrast objects is not sensitive to optical errors, it is obvious that the common practice of testing telescopes and binoculars with charts consisting black and white bars is a poor test of optical quality. Conclusions drawn on the basis of such charts do little to predict the performance of a binocular or telescope on objects with a low intrinsic contrast. Test charts with dark grey and light gray lines are more suitable for testing the performance of these instruments. The Paterson Optical Test Target designed by Geoffrey Crawly, editor of the British Journal of Photography, may then be an appropriate choice for resolution tests. It consists of 63 segments (each with structural details of different shape and ranging from course to very fine) in checkerboard fashion in black, grey and three colors.

Rutten and Van Venrooij published estimates of the loss of resolving power at different contrast levels and they came up with the values listed in their table below:

Contrast (%)	89	70	49	38	27	16	10	6	2,5
Resolving power (arcseconds)	74	76	80	82	84	89	95	105	127

Apart from the above described limitations for determining the resolution of an optical system one has also to take into account the circumstances that influence the acuity of the eye and the observation process.

The eye's resolution at full daylight amounts to approximately 1 minute of arc, but it decreases with increasing age. The resolution decline starts already from the age of 20 and at the age of 60 it is diminished by about 25%. That means that a 60 year old eye may have a visual resolution of around 75 arcseconds. The resolution is also diminished when binoculars are used handheld. Handheld binoculars with 8x magnification may perform 20-30% less due to muscular tremble or shake than supported binoculars and this difference grows with increasing magnification. This muscular tremble does affect the visual resolution.

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